Axial Structure of Gas Discharge Sustained by the Quadrupolar Wave in Three Component Magnetized Waveguide Structure

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The report presents the results of theoretical research of plasma density axial structure in gas discharge sustained by the travelling slightly–damping nonpotential quadrupolar surface wave (SW) with azimuthal wave number $m = \pm 2$ in diffusion controlled discharge regime. SW propagates in three components waveguide structure that consists of a cylindrical magnetized plasma column with radius $R$ and waveguide metal wall with radius $R_{cw} > R$. There is a vacuum gap between the plasma column and the metal wall. External constant magnetic field $\vec{B}$ is directed along the axis of the waveguide system. The quadrupolar mode was considered due to the fact that this wave is useful for gas discharge sustaining [1]. Plasma density axial distribution along the discharge was studied in the framework of the electrodynamic approach [2]. Such description has shown its efficiency for theoretical modeling of the long discharges sustained by the symmetric ($m = 0$) and dipolar modes ($m = \pm 1$) (see review paper [3]). The dispersion properties of quadrupolar waves and plasma density axial structure in simplified plasma–vacuum discharge structure were considered in [4]. The aim of this paper is to study the influence of waveguide metal wall presence on the dispersion properties of quadrupolar waves and on the gas discharge axial structure under various gas discharge parameters (external magnetic field, waveguide metal wall and plasma column radii). The discharge model considered, consists of the SW local dispersion equation, SW energy balance equation along the discharge, equation that connects together the SW power and the local plasma density [2]. The used method for numerical research of the basic system of equations, mentioned above, allows us to investigate gas discharges sustained by the SW with any azimuthal wave number.

The plasma is considered in hydrodynamical approximation as cold weakly absorbing media [5]. It is characterized by the effective electron collision frequency $\nu$ ($\nu << \omega$, $\omega$ is the SW generator frequency). It is assumed that this frequency $\nu$ is constant in discharge volume. Plasma density is considered to be homogeneous in radial direction and vary slightly along the plasma column.

The single-mode regime is considered, when only one nonpotential eigen wave is excited in discharge structure. First let us consider the influence of magnetic field value, plasma column and metal waveguide wall radii on the dispersion properties of quadrupolar waves. The results of the numerical investigation of the dispersion equation at various discharge parameters are presented in Figs.1-3. Dispersion properties of the quadrupolar SW with $m = -2$ at different external magnetic field values $\Omega = \omega_{ce} \omega^{-1}$ ($\omega_{ce}$ is electron cyclotron frequency) are presented.

856
When external magnetic field is strong ($\Omega > 1$) the dispersion of quadrupolar mode $m = -2$ becomes like zoned type (similar to symmetric and dipolar modes [2]). The zero dispersion zone corresponds to minimum possible value of the dimensionless axial wave number $k_3 R$. The number of dispersion zone increases with growth of minimum $k_3 R$ value. The SW with $m = -2$ from the zero dispersion zone possesses only the straight dispersion. In the first dispersion zone and in higher ones SW has the regions both of the straight and backward dispersion (curves 5,6 in Fig.1).

The effectiveness of quadrupolar SW used for the discharge sustaining can be evaluate from the SW damping rate $\delta = \text{Im}(k_3 R)$ dependence on the wavelength and local plasma density. Damping rate for SW with different azimuthal wavenumber values as a function of dimensionless wavenumber $\text{Re}(k_3 R)$ is shown in Fig.2. In the region of small values of dimensionless wavenumber the SW collisional damping increases with the growth of absolute value of azimuthal wavenumber ($|m|$). It is shown that damping rate of quadrupolar mode with $m = -2$ decreases with the increase of $\eta$ value. The increase of external magnetic field value of $\Omega$ results to the weakening of the dependence of $\delta$ on the wavelength.

### Figure 1
*Dimensionless wave frequency $\omega \omega_p^{-1}$ of quadrupolar ($m = -2$) mode as a function of dimensionless wavenumber $k_3 R$ at $\sigma = \omega R c^{-1} = 0.5$, $\eta = R_{\text{per}} R^{-1} = 1.1$. Line marked by the number 1,2,3,4,5,6 corresponds to value of $\Omega$: 0.2, 0.6, 1.2, 3.0 (zero zone), 3.0 (first zone), 3.0 (second zone, respectively).*

### Figure 2
*Dimensionless damping rate as a function of dimensionless wavenumber $\text{Re}(k_3 R)$ at $\sigma = 0.5$, $\Omega = 0.6$, $\eta = 1.5$, $\nu \omega^{-1} = 10^{-3}$. Line marked by the number 1, 2, 3, 4 correspond to the value of $m$ 0, -1, -2, -3, respectively.*

### Figure 3
*Dimensionless damping rate as a function of dimensionless wavenumber $\text{Re}(k_3 R)$ at $m = -2$, $\sigma = 0.5$, $\Omega = 0.6$, $\nu \omega^{-1} = 10^{-3}$. Line marked by the number 1, 2, 3, 4, 5 correspond to the value of $\eta$ 1.1, 1.2, 1.3, 1.5,3.0, respectively.*
The influence of metal radius value $R_{metal}$ on the dispersion properties of the quadrupolar wave with $m = -2$ at the fixed dimensionless plasma radius $\sigma = \omega R c^{-1}$ ($c$ is light velocity in vacuum) and dimensionless magnetic field value $\Omega = \omega R c^{-1}$ is presented in Fig. 4. The calculations show that the influence of metal wall radius on the SW dispersion properties is negligible when $\eta \geq 3.0$. The influence of waveguide metal wall radius on the dispersion properties of quadrupolar SW increases with the growth of external magnetic field value.

The influence of metal wall radius on the dispersion properties of quadrupolar mode with $m = 2$ for weak magnetic field ($\Omega = 0.6$) and fixed plasma column radius is also investigated (Fig. 5). The decrease of vacuum gap value leads to the decrease of SW phase velocity and also to the change of the dispersion nature. For narrow vacuum gaps ($\eta \leq 1.2$) the dispersion becomes straight type but SW group velocity is rather smaller. The influence of parameter $\eta$ is substantial when area of axial wavenumbers $k_zR$ is smaller and the wave has maximum possible value of phase and grope velocities.

Let us consider the discharges sustained by quadrupolar SW with azimuthal wave number $m = -2$ in diffusion controlled regime. The axial distribution of dimensionless plasma density $N = \omega R c^{-2}$ is studied from the generator exit up to the end of the discharge. The dimensionless plasma density $N$ value just near to the generator was determined on the basis of SW dispersion properties. The discharge length $L$ is defined by such dimensionless axial coordinate $\xi = \nu z(\omega R)^{-1}$ value ($z$ is axial coordinate counted from the generator), where the total SW energy flux turns to zero. The plasma density axial structure in the discharge sustained by the SW with $m = -2$ at dimensionless radius $\sigma = 0.5$ and weak external magnetic field value for several $\eta$ parameters is shown in Fig. 6. The decrease of a vacuum gap value leads to the substantial growth of maximum possible plasma density value in the discharge. The external magnetic field value greatly affects the axial distribution of

![Figure 4](image1.png)

*Dimensionless wave frequency of quadrupolar mode with $m = -2$ as a function of dimensionless wavenumber $k_zR$ at $\sigma = 0.5$ and $\Omega = 0.6$ for various metal waveguide radius values. Line marked by the number 1, 2, 3, 4, 5 correspond to the value of $\eta$ 1.1, 1.2, 1.3, 1.5, 3.0, respectively.*

![Figure 5](image2.png)

*Dimensionless wave frequency of quadrupolar mode with $m = 2$ as a function of dimensionless wavenumber $k_zR$ at $\sigma = 0.5$ and $\Omega = 0.6$ for various values of the metal waveguide radius. Line marked by the number 1, 2, 3, 4, 5 correspond to the value of $\eta$ 1.1, 1.2, 1.3, 1.5, 3.0, respectively.*
plasma density. The increase of external magnetic value results to the increase of possible plasma density in the discharge (Fig.7).

The increase of plasma column radius leads to the small-scale plasma density growth.

![Figure 6](image_url)  
**Figure 6** Axial normalized plasma density profiles of plasma columns sustained quadrupolar mode with \( m = -2 \) at \( \sigma = 0.5 \) and \( \Omega = 0.6 \) for different metal waveguide radius values. Line marked by the number 1, 2, 3, 4, 5 correspond to the value of \( \eta \) 1.1, 1.3, 1.5, 1.7, 1.9, respectively.

The decrease of the vacuum gap value can greatly change the nature of the dispersion of quadrupolar mode with \( m = 2 \) (Fig.5) Calculations of plasma density axial structure have shown, that the discharge lengths in this case are extremely small.

The plasma density axial distribution, SW field radial structure and SW power were investigated for different values of external magnetic field, plasma column and waveguide metal wall radii. It was find, that variation of waveguide metal wall radius substantially affects the SW dispersion properties, SW spatial damping and SW radial wave field distribution as well as the electron density axial distribution. Carried out investigations allows to compare spatial distribution of gas discharge parameters for any SW azimuthal wave number and to find the conditions, under which rather homogeneous axial plasma density profile may be obtained.

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**References**


