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The application of an Eulerian Vlasov code for the study of kinetic effects in an inductively coupled discharge

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An Eulerian Vlasov numerical code has been developed for the numerical simulation of colionsless heating by the anomalous skin effect in an inductively coupled discharge. The code is one-dimensional (1D) and applies a method of fractional steps for the solution of the Vlasov-Maxwell kinetic equations[1]. Eulerian Vlasov codes have a very low noise level, and consequently allow an accurate simulation of the heating of the electron distribution function.

The system is supposed to be one-dimensional in x. The electrons are reflected at x = 0 to model the presence of an electrostatic sheath potential, and, to preserve neutrality, ions are also reflected. An RF coil excites an inductive electromagnetic field at the x = 0 boundary. This electromagnetic field which penetrates the plasma has the components $\vec{E} = E_y(x, t)\,\overrightarrow{e_y}$, $\vec{B} = \overrightarrow{B_z}(x, t)\,\overrightarrow{e_z}$. E_y and B_z satisfy Maxwell's equation: $\dfrac{\partial E_y}{\partial x} = -\dfrac{\partial B_z}{\partial t}$.

We *use* normalized units, whose relation to CGS-Gaussian ones is as follows:

$E_y, B_z, F^{\pm}\,(\omega v_t m_e / e);\ A_y\left(v_t^2\ m_e / e\right);\ x(v_t / \omega);\ N_{e,i}(n_o);\ v(v_t);\ f(v_t);\ (n_o / v_t);\ \varphi(T_e / e);\ P(m_e v_t)$

where n_o and v_t are the electron density and electron thermal velocity of the initially uniform plasma, and ω is the angular frequency of the RF field.

As the Hamiltorian has no dependence on y, the canonical momentum for the electrons

$$P_y = v_y - \frac{v_t}{c} A_y \qquad \left(\frac{m_i}{m_e} v_y + \frac{v_t}{c} A_y \quad \text{for the ions} \right) \qquad (1)$$

is a constant of the motion, where $A_y(x,t)$ is the vector potential, which may be obtained from

$$\frac{v_t}{c}\frac{\partial A_y}{\partial t} = -E_y, \qquad \frac{\partial A_y}{\partial x} = -B_z \qquad (2)$$

The kinetic equation for the electrons is the one-dimensional Vlasov equation:

$$\frac{\partial f_e}{\partial t} + v_x \frac{\partial f_e}{\partial x} - \left(E_x + \frac{v_t}{c} u_{ye} \; B_z \right) \frac{\partial f_e}{\partial v_x} = 0 \tag{3}$$

with a similar equation for the ions' distribution function f_i with a factor m_e / m_i added in front of the last parenthesis and a change in the sign, and u_{yi} replacing u_{ye} We assume that the plasma is cold in the transverse direction *y* and that the *y* velocity components of electrons and ions obey their respective fluid equations (in view of Eqs (1-2)):

$$\frac{\partial u_{ye,i}}{\partial t} = \begin{cases} - E_y & \text{electrons} \\ + \dfrac{m_e}{m_i} E_y & \text{ions} \end{cases} \tag{4}$$

Maxwell's equations are written in terms of $F^{\pm} = E_y \pm B_z$

$$\left(\frac{\partial}{\partial t} \pm \frac{c}{v_t} \frac{\partial}{\partial x} \right) E^{\pm} = \frac{\varpi_{pe}^2}{\varpi^2} \left(n_e \, u_{ye} - n_i \, u_{yi} \right) \tag{5}$$

and E_x is calculated from Poisson’s equation:

$$\frac{\partial^2 \phi}{\partial x^2} = - \frac{\omega_{pe}^2}{\omega^2} \left(n_i - n_e \right); \quad E_x = - \frac{\partial \phi}{\partial x} \quad \text{with } n_{i,e} = \int f_{i,\,e}(x, v_x) \, dv_x \;\; \text{and } \omega_{pe}^2 = \frac{4\pi \; n_o e^2}{m_e} \tag{6}$$

We note in Eqs. (5) and (6) the factor $\omega_{pe}^2 / \omega^2 = \left(v_t^2 / \omega^2 \right) / \lambda_{De}^2$, i.e. the ratio of the distance a thermal electron travels in a RF period (divided by 2π) to Debye length. This ratio ω_{pe}^2 / ω^2 is large for an RF wave; therefore, a high resolution in time and space is necessary to accurately calculate the current contribution in Eq. (5) and the charge in Eq. (6). The solution for the charge separation in Eq. (6) requires $\Delta x / \lambda_{De} \leq 1$. We chose the normalized time step $\Delta t = \frac{v_t}{c} \Delta x$ so that the left-hand side in Eq. (5) is shifted from grid point to grid point without having to do any interpolation[1]. At $x = 0$, we apply a RF signal $F^{+} = 2 \, E_o \sin \omega t$ corresponding to a field entering the plasma.

Let us define $t_n = n \, \Delta t$, where Δt is the time-step. The velocity space is divided into N_v, grid points between $- v_{\max}$ and $+ v_{\max}$. The length L of the 1D system is divided into N_x

grid points. The Vlasov equation is advanced in time using a fractional steps or splitting technique[1,2].

We chose to simulate, with our Eulerian Vlasov code, the inductive heating of a hydrogen plasma with parameters similar to those of Turner's simulation who used a Particle-in-Cell (PIC) code[4]. In his simulation, Turner did not include the electrostatic field (E_x), which is often neglected in PIC simulations of discharges, possibly due to statistical noise problems. In our work, we made comparative runs, with and without the electrostatic field. We took $T_e = 5$ eV, $n_c = 10^{12}$ cm^{-3}, and f = 13.56 MHz. Hence $v_t / c = 1.42\times10^{-3}\sqrt{T_e} = 3.18\times10^{-3}$, $\omega = 2\pi\times13.56\times10^{6}\ s^{-1}$. Thus, in our normalized units $n_e = 10^{12} / cm^3$. Hence $\omega / \omega_{pe} = 1.51\times10^{-3}$. In our normalized units $\lambda_{De} /(v_t / \omega) \equiv \omega / \omega_{pe} = 1.51\times10^{-3}$. We take a length L= 4 (i.e. 4.4 cm). With N= 4200 points in space, we have $\Delta x = L / N = 0.95\times10^{-3} = 0.63\ \lambda_{De} /(v_{th} / \omega)$, which satisfies the usual stability criteria. The amplitude of the incident field was about 2 Volts/cm. In Figures 1, we show the transverse electric field, and in Fig. 3, the transverse current during one period of the wave oscillation. It is seen that (Fig. 2) the decay of the electric and magnetic fields closely follows the usual law for the ordinary skin effect, $\exp(= x / \delta_s)$, with the scale length equal to the usual skin depth $\delta_s = c / \omega_{pe}$. This is true for the runs with and without the axial field E_x. The axial current and the electron and ion density modulations are the items which differ most between these runs. This axial current is at the harmonic frequency 2ω. We will present these results in the light of the analytical calculations of Cohen and Rognlien[5], and of recent measurements of J_x at 2ω by Godyak *et al.*[6]. Figure 4 represents the magnetic field B_z of the wave during one period of the wave oscillation. The heating term $J_y\ E_y$ is represented in Fig. 5, and the force term $u_{ye}\ B_z$ is represented in Fig. 6.

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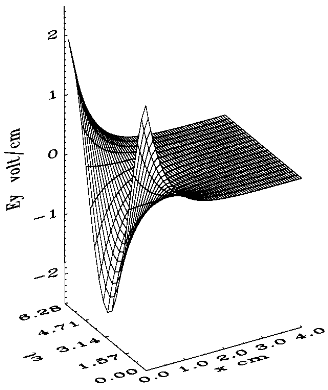

Fig. 1 Space-time profile of the transverse electric field E_y.

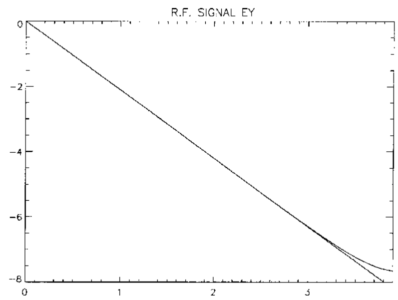

Fig. 2 The amplitude of the transverse electric field closely matches a simple exponential with scale length $\delta = c/\omega_{pe}$.

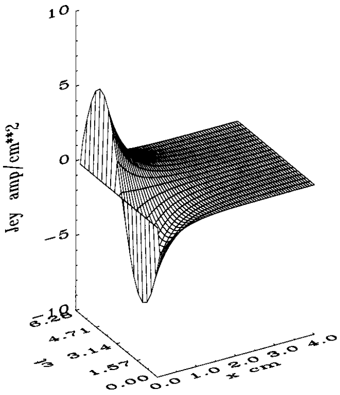

Fig. 3 Space-time profile of the transverse current.

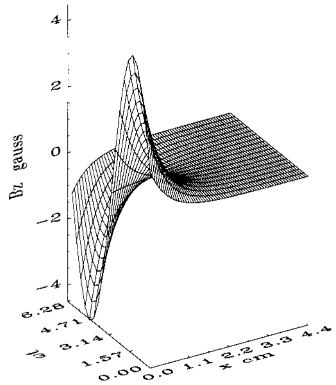

Fig. 4 Space-time profile of the wave magnetic field B_z.

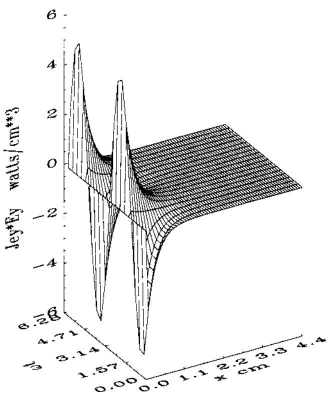

Fig. 5 Space-time profile of the heating term J_yE_y.

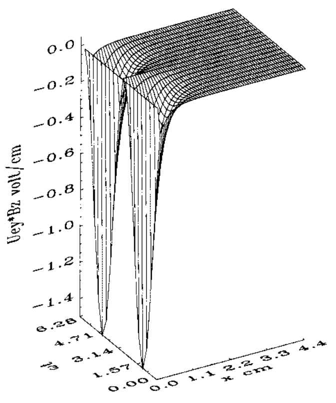

Fig. 6 Space-time profile of the force term u_{ye} B_z