On the problem of the accuracy of Fast Wave absorption estimate at the fundamental ICR harmonic.

S. S. Pavlov* and F. Castejón**

*Institute of Plasma Physics. Kharkov Institue of Physics and Techology (Kharkov, Ukraine)

**Asociación Euratom-Ciemat para Fusión. (Madrid, Spain)

1.- Introduction.

It is known that some unphysical effects can be obtained when Fast Wave (FW) propagation and absorption at the fundamental harmonic is studied numerically. The dispersion equation is solved performing a series development up to the second order in the finite Larmor radius parameter: $k_\perp \rho_i$ ($k_\perp$ is the perpendicular wave number and ρ_i is the ion Larmor radius). This approximation is used both in ray tracing theory and solving the wave equation in the weak space dispersion limit.

The main unphysical effect that appears is the negative disipation of energy at high frequency. This effect is apparently connected with the finiteness of the expansion and may cause some doubts on the reliability of the calculations. The works [1,2] are totally devoted to study this problem and, in particular, deal with the plasma conditions for which negative disipation arises. It was found that the main parameter that governs the apparison of negative disipation was the concentration of resonant ions, N_i: the negative dissipation appears when the concentration becomes high enough.

Several studies have been performed in the context of doing calculations avoiding the problem of negative dissipation. Those studies play with different variants of the "hot" terms of dispersion relation, the terms that are connected with power absorption at the fundamental ICR frequency. The fully omission of these terms gave the so called "SCK"-form [3,4]. Secondly; some forms in which only imaginary parts of the terms are omitted. And a third group exists that perform the phenomenological modification of these terms, justified by numerical calculations. However, all these approximations can be questioned in the case of quasiperpendicular propagation of FW, when $N_\parallel$ goes to 0.

It is known that the expansion of FW dispersion equation on the parameter $k_\perp \rho_i$ converges in different ways for different propagation angles with respect to magnetic field. In particular, in the case of quasiperpendicular propagation the expansion converges essentially slower than in the case of other angles. Therefore, more terms can be necessary in the case of the calculation of propagation and absorption at first harmonic of ICR.

Consequently, the value of logitudinal refractive index, $N_\parallel$, may be proved to be the main parameter of localization of negative dissipation in a plasma. This work is devoted to the study of the influence of the strong space dispersion upon absorption of FW in the range of fundamental ICR. This task is also proved to be closely connected with the problem of negative dissipation.

2.- Calculation of the FW absorption with strong space dispersion.

We shall look for the absorption of FW by means of the solution of dispersion equation, that connects components of dielectric tensor with refractive index. The full dispersion equation describing FW, SW, and IBW has the form:

$$\epsilon_{11}N_\perp^2 + 2N_\parallel\epsilon_{13}N_\perp^3 + \left[-\epsilon_{11}(\epsilon_{22}-N_\parallel^2) - \epsilon_{33}(\epsilon_{11}-N_\parallel^2) + \epsilon_{13}^2 - \epsilon_{12}^2\right] N_\perp^2 +$$

$$2N_\parallel\left[\epsilon_{12}\epsilon_{23} - (\epsilon_{22}-N_\parallel^2)\epsilon_{13}\right] N_\perp + \epsilon_{33}\left[(\epsilon_{11}-N_\parallel^2)(\epsilon_{22}-N_\parallel^2) + \epsilon_{12}^2\right]$$

$$+2\epsilon_{12}\epsilon_{23}\epsilon_{13} - (\epsilon_{22}-N_\parallel^2)\epsilon_{13}^2 + (\epsilon_{11}-N_\parallel^2)\epsilon_{23}^2 = 0\ ,(1)$$

where ϵ_{ik} are the components of dielectric tensor that can be written as expansions in the parameter $(k_\perp\rho_i)^2$.

The equation (1) can be transformed into a double quadratic equation by taking into account that ϵ_{13} and ϵ_{23} are proportional to $N_\perp$. We shall denote $\epsilon_{13}^* = \epsilon_{13}/N_\perp, \epsilon_{23}^* = \epsilon_{23}/N_\perp$. The FW and SW branches are easily detached in the ICRF range because of the relation $\epsilon_{33} \gg \epsilon_{11}, \epsilon_{12}, \epsilon_{22}$. As a consequence, the usual equation for this branch in the ICR frequency range can be used to calculate the FW absorption:

$$(\epsilon_{11}-N_\parallel^2)N_\perp^2 - (\epsilon_{11}-N_\parallel^2)(\epsilon_{22}-N_\parallel^2) - \epsilon_{12}^2 = 0, (2)$$

The solution of this equation can be found, of course, numerically but, moreover, one can deal with it analitically. Considering the size of the several terms that appear in (3) and doing some approximations, as keeping only the main terms, we obtain the following dispersion equation for FW which is valid for any value of $N_\parallel \geq (k_\perp\rho_i)^2$:

$$N_\perp^4\left(-\frac{3}{4}i\frac{\omega_{pi}^2}{\omega^2}\sqrt{\pi}Z_0\rho_i^4\frac{\omega^4}{C^4}W_{-1}\right) + N_\perp^2 - (N_F^0)^2 = 0, (3)$$

where $(N_F^0)^2 = \frac{(\epsilon_1^0-\epsilon_2^0-N_\parallel^2)(\epsilon_1^0+\epsilon_2^0-N_\parallel^2)}{\epsilon_1^0-N_\parallel^2}$ is the square of refractive index for FW in the "cold" approximation. The roots of equation (3) describe FW properties (exactly for $N_\parallel \geq (k_\perp\rho_i)^2$) and any Bernstein wave (with less level of accuracy). For FW we will have:

$$ImN_\perp^2 = \frac{(X-1)ReW_{-1} - YImW_{-1}}{d\left|W_{-1}\right|^2},$$

$$ReN_\perp^2 = \frac{(X-1)ImW_{-1} + YReW_{-1}}{d\left|W_{-1}\right|^2}, (4)$$

where we have defined the quantities $d = 3\sqrt{\pi}N_A^2 Z_0\frac{\omega^4}{C^4}\rho_i^4; N_A^2 = \frac{\omega_{pi}^2}{\omega_{Ci}^2};$

$$X = [\frac{1}{2}([1+d^2\left|(N_F^0)^2\right|^2\left|W_{-1}\right|^2 + 2d(Im(N_F^0)^2ReW_{-1} + Re(N_F^0)^2ImW_{-1})]^{1/2} + 1+$$

$$d(Im(N_F^0)^2ReW_{-1} + Re(N_F^0)^2ImW_{-1}))]^{1/2},$$

$$Y = [\frac{1}{2}([1 + d^2 \left|(N_F^0)^2\right|^2 |W_{-1}|^2 + 2d(Im(N_F^0)^2 ReW_{-1} + Re(N_F^0)^2 ImW_{-1})]^{1/2} - 1-$$
$$d(Im(N_F^0)^2 ReW_{-1} + Re(N_F^0)^2 ImW_{-1}))]^{1/2}.$$

These expressions describe the dispersion and absorption of FW for any value of $N_\parallel \geq (k_\perp \rho_i)^2$ near the resonance $\omega = \omega_{ci}$. For instance, we can consider exactly the resonance case, where $ReW_{-1} = 1$ and $ImW_{-1} = 0$. We shall have the following possibilities:

1) $N_\parallel \gg 1$ $(d \ll 1)$ $ImN_\perp^2 = Im(N_F^0)^2 + \frac{3}{4}\left|(N_F^0)^2\right|^2 \sqrt{\pi} N_A^2 Z_0 \frac{\omega^4}{C^4}\rho_i^4$ $ReN_\perp^2 = Re(N_F^0)^2$

2) $N_\parallel \ll 1$ $(d \gg 1)$ $ImN_\perp^2 = ReN_\perp^2 \simeq \frac{1}{\sqrt{2}}\frac{\left|(N_F^0)^2\right|^{1/2}}{d^{1/2}}$, (5)

For the plasma parameters typical of "T-10" tokamak (B_0=3T, $n_e = 7 * 10^{13} cm^{-3}$, $T_i = T_e = 3KeV$, one obtains that the absorption is enhanced about 5 times as comparied with "cold" case at $N_\parallel = 3$. And at $N_\parallel = 5$ the values of full absorption and the "cold" one are equal. When the value of $N_\parallel$ decreases the absorption sharply increases and at about $N_\parallel = 0.1$ one has $ImN_\perp^2 \simeq ReN_\perp^2$. Furthermore, when the value of $N_\parallel$ goes to 0, then $ImN_\perp^2$ goes to 0 too, in accordance with expression (5). To evaluate the increasing of absorption that this calculations provide, it is useful to compute the optical thickness $\tau = \int Imk_\perp dR$. This quantity is estimated assuming that magnetic field B_0 changes as $1/R$ and ICR absorption is proved to be strong only in a narrow domain $\Delta R = R_0/Z_0$.

For the case of cold plasma, we have [1]

$\tau = Im(k_\perp \Delta R) = ReK_\perp \Delta R/(4\sqrt{\pi}Z_0) = k_A R_0/(4\sqrt{\pi}Z_0^2)$, (6)

For the case of "hot" plasma it is necessary to use the expression (4) to calculate $Imk_\perp$. Then we have

$\tau = Imk_\perp \Delta R = \frac{1}{2}\frac{ImN_\perp^2}{ReN_\perp^2} k_A \frac{R_0}{Z_0} = \frac{1}{2}\frac{X-1}{2Y} k_A \frac{R_0}{Z_0}$, (7)

This formula is simpler for the extreme cases:

a) $N_\parallel \gg 1 (d \ll 1), \tau = k_A(3\sqrt{\pi}/8) R_0 k^4 \rho_i^4 + k_A \frac{R_0}{4\sqrt{\pi}Z_0^2}$, (8)

b) $N_\parallel \ll 1 (d \gg 1), \tau = k_A R_0/(2Z_0) + k_A \frac{R_0}{4\sqrt{\pi}Z_0^2}$, (9)

Thus when $N_\parallel \gg 1$, the optical thickness does not depend on $N_\parallel$ and therefore, it may be estimated using formula (6) for "cold" plasma at the value of $N_\parallel$ for which the full absorption and "cold" one are approximately the same. This value of $N_\parallel$ is easy to obtain from the equation $Z_0(k_\perp \rho_i)^2 = 1$. The calculations for the same plasma as before give for a middle device of the type of tokamak "T-10" a small value of optical thickness $\tau = 10^{-4}$. However for large devices of the type of "ITER" tokamak one has $\tau = 0.5$ and therefore, RF heating by FW at first harmonic resonance becomes rather interesting as a heating regime. Note also the fact that the optical thickness does not depend on $N_\parallel$, except for very small values of $N_\parallel$, makes easier the task of injecting high frequency power into plasma due to the possibility of using smaller values of $N_\parallel$.

3.- The physical nature of the "additional" FW absorption.

To find out the nature of appearing of the FW additional absorption in the case of quasiperpendicular propagation, let us calculate FW polarisation in "cold" and "hot" cases. In the "cold" case, it can be readily obtained from the wave equation:

$$\frac{E_Y}{E_X} = \frac{\epsilon_1 - N_\parallel^2}{i\epsilon_2} \approx i\left(-1 + \frac{3}{2\pi Z_0^2}\right) - \frac{1}{Z_0\sqrt{\pi}} \approx (-i) - \frac{1}{\sqrt{\pi}Z_0}$$

where the value of imaginary part corresponds to circular polarisation of FW and its sign corresponds to the direction of rotation of the electrical field vector, against the direction of ion rotation. Since one has that $Z_0 \gg 1$, the real part of the polarisation introduces a very weak ellipticity and is the responsible of the existence of very weak absorption $\sim \left|\frac{E_Y}{E_X}\right|^2 \sim \frac{1}{\pi Z_0^2}$.

In the "hot" case the polarisation is given by:

$$\frac{E_Y}{E_X} \approx i\left(-1 - x + ...\right) - \frac{1}{2\sqrt{\pi}Z_0\left(\frac{1}{2} - x + ...\right)}$$

where $x = (k_\perp \rho_i)^2$, $A_1(x) = \exp(-x)I_1(x)$, and I_1 is the modified Bessel function of order 1. From this expression it follows that in the real part there are not practically changes in comparison with the "cold" case, because $x \ll 1$ for FW. However in the imaginary part it appears a small addendum which is proportional to x. This term introduces additional ellipticity in the polarisation and, consequently, enhances the FW absorption. This effect is due to the fact that the absorption is proportional to $\sim \left|\frac{E_Y}{E_X}\right|^2$, then the absorption strengthening must be $\sim x^2 = (k_\perp \rho_i)^4$. Comparison of the additional "hot" absorption with "cold" one gives

$$\frac{D_{hot}}{D_{cold}} = x^2 \pi Z_0 = \pi \left(k_\perp \rho_i\right)^4 Z_0^2.$$

From that appreciation it follows that when $N_\parallel$ decreases the additional "hot" absorption may become stronger than the "cold" one if $\pi \left(k_\perp \rho_i\right)^4 Z_0^2 > 1$ is satisfied. Thus, if the value of $N_\parallel$ is small enough the "hot" addition in the polarization will enhance the FW absorption. The calculation of this absorption using the more strict formula gives, in terms of the main parameter $x = (k_\perp \rho_i)^2$, the following result

$$D = \frac{\omega}{8\pi}\left|E_X\right|^2 \sqrt{\pi} N_A^2 Z_0 \exp(-Z_{-1}^2)\left[\frac{x^2}{4} + \frac{1}{2\pi Z_0^2(\exp(-Z_{-1}^2))^2}\right]$$

The first term corresponds to "hot" absorption and the second term with "cold" one.

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