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Calculation of Radial Electric Field and Poloidal Rotation in Tokamak Edge Plasmas under Consideration of Anomalous Transport

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1.Introduction

In a recent paper [1] the magnetic surface averaged values of the radial electric field $< E_r >$ and the $\|\vec{B}$ center-of-mass velocity $< U_{\|} >$ of the plasma have been calculated for prescribed radial density and temperature profiles using the total momentum balance within a magnetic flux surface under the global ambipolarity constraint $< j_r >= 0$ combined with particle, radial momentum and heat balance equations of the individual plasma constituents. The equations have been derived and numerically solved for a pure hydrogen plasma within a revised neoclassical transport model in the Pfirsch-Schlüter regime, accounting for inertia and gyroviscous forces (FLR-effects) and for corrections in the viscous forces related to longitudinal velocity and heat flux gradients. All of these forces are affected by the redistribution of the plasma parameters within a magnetic flux surface due to strong electric drift motion competing with $\|\vec{B}$ diffusion processes in a dynamic equilibrium with the centrifugal and $\vec{\nabla}B$ drifts (as driving mechanism) and cause the transition from automatic to forced global ambipolarity by the (nonlinear) appearance of $< E_r >$ and $< U_{\|} >$ in the governing equations. The latter have been transformed into a set of two nonlinearly coupled first order ordinary differential equations and solved numerically. Like 2D edge models [2] the present 1D model follows [3] by including anomalous radial diffusion (still assuming anomalous transport to be ambipolar) and anomalous (instead of classical collisional) viscous shear forces caused by radial gradients of both $< U_{\|} >$ and the ion drift motion within a magnetic flux surface. In contrast to collisional ones anomalous viscous shear forces are not negligibly small compared to gyroviscous and inertia forces in the B-weighted parallel momentum balance competing with forces due to longitudinal viscosity effects and may even dominate the global ambipolarity constraint. As a consequence the strong coupling between $< E_r >$ and $< U_{\|} >$ via the standard nc coupling between poloidal velocity and radial ion temperature gradient is relaxed. The problem is now governed by a set of two nonlinearly coupled second order ordinary differential equations with a corresponding higher flexibility in the boundary conditions. Then solutions are possible with a rather smooth radial change of $U_{\|}$ and a corresponding strong coupling between binormal and poloidal plasma rotation. In particular, by reducing the anomalous transport coefficients below typical L-mode values, solutions with strong shear of both radial electric field and poloidal velocity appear, passing extrema within a thin edge layer as seen in the H-mode.

2. Governing equations

As in the revised purely neoclassical case we determine the magnetic surface averaged values $< E_r >$ and $< U_{\|} >$ from the ambipolarity constraint based on the toroidal momentum equation and a reduced form of the B-weighted parallel momentum balance

$$< j_r >_\phi = < \frac{\vec{e}_\phi}{\Theta B} \bullet \vec{M} > = 0 \quad ; \quad < \vec{B} \bullet \vec{M} > - < \Theta B^2 >< j_r >_\phi = 0 \qquad (1)$$

$$\vec{M} \equiv \sum_\alpha \left[\vec{I}_\alpha - m_\alpha n_\alpha (\vec{u}_\alpha \bullet \vec{\nabla})\, \vec{u}_\alpha - \vec{\nabla} \bullet \overleftrightarrow{\Pi}_\alpha \right] \quad ; \quad \vec{I}_\alpha = \int m_\alpha (\vec{v} - \vec{u}_\alpha) \left. \frac{\partial_s f_\alpha}{\partial \tau} \right|_{inel.} d^3 v$$

the sum being performed over all ion components α neglecting electron contributions to momentum sources $\vec{I}_\alpha$, inertia and viscosity. The poloidal harmonics of the total plasma pressure $p^\Sigma = \sum_\alpha p_\alpha + p_e$ follow directly from the ambipolarity constraint based on the binormal momentum component

$$< j_r >_\perp = < \frac{\vec{e}_\perp}{B} \bullet \left[\vec{M} - \vec{\nabla} p^\Sigma\right] > = 0 \qquad (2)$$

3. Formulation for a turbulent two-component plasma

Eqs.(1-2) have been evaluated for a turbulent highly collisional two-component (pure) edge plasma using the standard magnetic field $\vec{B} = \left[0, -\Theta(r), 1\right] R_0 B_{\phi 0}/R$ with $R/R_0 = 1 + r\cos\theta/R_0$, $RB_\phi = const$, $\Theta^2 \ll 1$ and $< .. > = \oint Rd\theta/(2\pi R_0)..$, decomposing the plasma parameters according to $Q(r,\theta) \simeq Q_0(r) + Q_1^s(r)\ \sin\theta + Q_1^c(r)\ \cos\theta$ by neglecting higher harmonics in the poloidal Fourier spectrum (for purely nc-effects in a more general $\vec{B}$ geometry see [4]). For standard $\vec{B}$ eqs.(1-2) yield $\hat{p}_1^{\Sigma s} \simeq\ -2r/(\Theta p_0)\ \oint d\theta/(2\pi)\ \cos\theta\, M_{||}$ for the $\sin\theta$ amplitude of the normalized total pressure $\hat{p}_1^\Sigma = p_1^\Sigma/p_0$. Although this result reconciles $< j_r >_\perp = 0$ with $< j_r >_\phi = 0$, it has a negligible influence on the poloidal plasma variation provided that $(\rho_{p0}/\Delta)^2 \ll 1$ holds, where $\rho_{p0} = v_{T0}/(\Theta\Omega_0)$ with $v_{T0} = \sqrt{2kT_0/m}$ is the poloidal ion gyroradius and Δ is the radial gradient scale length of the edge plasma parameters. Likewise (in case of $\Omega_0 D_{\perp 0}^{an}/v_{T0}^2 \ll 1$ and weak poloidal dependence) anomalous transport affects preferentially eqs.(1-2), since its divergence in the particle and heat balance ($\sim \cos\theta$) is negligibly small compared to that of the vertical drift ($\sim \sin\theta$). In a pure hydrogen plasma the approximation $p_1^\Sigma(r,\theta) \simeq 0$ together with the assumptions of a large $\|\vec{B}$ electron heat conductivity and a $\|\vec{B}$ electron Boltzmann equilibrium reduce the set of supplementary equations to the ion heat balance with a simple collisional $\|\vec{B}$ heat transport relation [5]. In this case the nc-effects of eqs.(1-2) are governed by two characteristic parameters, which are usually $\leq O(1)$,

$$\Lambda^* = \frac{r}{\Delta\Theta^2\Omega_0\tau_0} = \frac{\rho_{p0}}{\Delta}\ \frac{qR_0}{\lambda_{mfp,0}^C} \qquad \text{and} \qquad \Lambda^{**} = \frac{R_0}{r}\ \frac{qR_0}{\sqrt{\lambda_{mfp,0}^C \lambda_{mfp,0}^X}}$$

where $\lambda_{mfp,0}^C = v_{T0}\tau_0$ and $\lambda_{mfp,0}^X = v_{T0}/\nu_{ia,0}^X$ are respectively the mean free paths for ion-ion Coulomb ($\tau_0 \equiv \tau_{ii,0}$) and ion-atom charge-exchange collisions. For order of magnitude estimates we adopt Bohm's scaling $\eta_{2,0}^{an} = mn_0 D_{\perp 0}^{an}$ with $D_{\perp 0}^{an} = (kT_0)/(f_B e B_0)$. Then, the first of eqs.(1), neglecting terms of order $\Theta^2 q^2 = r^2/R_0^2 \ll 1$, translates into

$$mn_0(\nu_{ea,0}^I + \nu_{ia,0}^X)u_{a\phi,0} - \underbrace{mn_0\nu_{ia,0}^X u_{\phi 0}}_{\leq O\left(\frac{125}{8}\ \frac{\Lambda^{**2}}{\Lambda^{*2}}\right)} - \underbrace{\frac{1}{5\Theta R_0}\ \frac{d}{dr}\left(\frac{p_0}{\Omega_0}\ \hat{n}_1^s u_{\theta 0}\right)}_{O(1)} + \qquad (3)$$

$$\underbrace{+ \frac{d}{dr}\left(\eta_{2,0}^{an}\ \frac{dU_{||0}}{dr} + \Theta\eta_{1,0}^{an}\ \frac{du_{\perp 0}}{dr}\right) + mD_{\perp 0}^{an}\frac{dn_0}{dr}\ \left(\frac{dU_{||0}}{dr} + 2q^2\Theta\ \frac{du_{\perp 0}}{dr}\right)}_{\leq O\left(\frac{125}{16}\ \frac{\Omega_0\tau_0}{q^2 f_B} \to \frac{75}{8q^2} \text{ for } f_B \to \frac{5}{6}\ \Omega_0\tau_0\right)} \simeq 0$$

where $O(1)$ is the reference term. Neither inertia nor longitudinal viscosity contribute to $< j_r >_\phi = 0$. The first two terms in the first line of eq.(3) describe the momentum transfer from atoms to ions by ionization and charge-exchange. The next term is due to gyroviscosity, whereas the last line accounts for anomalous diffusion and shear viscosity. $f_B \to (5/6)\Omega_0\tau_0$ marks the transition from anomalous Bohm-like to classical collisional shear viscosity $\eta_{2,0} = 6p_0/(5\Omega_0^2\tau_0)$ [1]. The second of eqs.(1) passes into

$$\underbrace{-3 < j_r >_{\perp}^{lv}}_{O(1)} + \underbrace{\frac{mn_0\hat{n}_1^s}{\Theta^2 B_0 R_0}\left[\frac{\Theta k T_0}{eB_0}\frac{dU_{\|0}}{dr} + \left(u_{\theta 0} - \frac{2kT_0}{eB_0}\frac{d\ln(n_0\sqrt{T_0})}{dr}\right)\frac{E_{r0}}{B_0} - \frac{\Theta^2 U_{\|0}^2}{2}\right]}_{O(\Lambda^{*2})} +$$

$$+ \underbrace{\frac{2q^2}{B_0}\frac{d}{dr}\left[\eta_{2,0}^{an}\frac{d}{dr}\Big(u_{\perp 0} - \frac{\Theta U_{\|0}}{2}\Big)\right] + \frac{2q^2 m D_{\perp 0}^{an}}{B_0}\frac{dn_0}{dr}\frac{d}{dr}\Big(u_{\perp 0} - \frac{\Theta U_{\|0}}{2}\Big)}_{O\left(\frac{r\Lambda^*}{\Delta f_B} \to \Theta^2\Lambda^{*2} \ll 1 \text{ for } f_B \to \frac{5}{6}\,\Omega_0\tau_0\right)} \simeq 0 \qquad (4)$$

The first term of eq.(4) stems from $\|\vec{B}$ viscosity $[< \vec{B}\bullet(\vec{\nabla}\bullet \overset{\leftrightarrow}{\Pi}_0) > = -3\Theta B_0^2 < j_r >_{\perp}^{lv}]$

$$< j_r >_{\perp}^{lv} = \frac{\eta_{0,0}}{3B_0 R_0 r}\frac{Q}{S}\,\hat{n}_1^s\left[\Big(\frac{8\check{x}}{5} - 1\Big)\,u_{\theta 0} + (3 - \hat{x} - 2\check{x})\,\frac{kT_0}{eB_0}\frac{d\ln(n_0\sqrt{T_0})}{dr}\right] + \qquad (5)$$

$$+ \frac{\eta_{0,0}}{2B_0 R_0^2}\Big(u_{\theta 0} + x\,\frac{kT_0}{eB_0}\frac{d\ln T_0}{dr}\Big) \quad ; \quad u_{\theta 0} = u_{\perp 0} - \Theta U_{\|0} = \frac{kT_0}{eB_0}\frac{d\ln p_0}{dr} - \frac{E_{r0}}{B_0} - \Theta U_{\|0}$$

$$\hat{n}_1^s = -\frac{16}{25}\frac{d\ln T_0/dr}{\Theta^2\Omega_0\tau_0}\frac{r^2/R_0}{1 + Q^2/S^2} \quad ; \quad \frac{Q}{S} = \frac{32}{25}\frac{r}{\Theta^2 v_{T0}^2\tau_0}\left[\frac{4}{5}\,u_{\theta 0} - \frac{kT_0}{eB_0}\frac{d\ln(n_0\sqrt{T_0})}{dr}\right]$$

Since both $R_0\hat{n}_1^s/r$ and Q/S are of order Λ^*, the first term of eq.(5) is of order Λ^{*2} compared to the second (standard nc) one. The various coefficients are calculated in Grad's 21 moment approximation: in particular $x \simeq 1.833$, $\check{x} \simeq 1.779$, and $\hat{x} \simeq 1.846$ and $\eta_{0,0} \simeq 0.960\,p_0\tau_0$. The second term in eq.(4) is due to the partially compensating classical inertia and gyroviscosity [1], whereas the remaining terms refer to anomalous diffusion and shear viscosity. In the derivation of eqs.(3,4) we made use of the edge plasma relations $2q^2 \gg 1$ (q = safety factor) and $d\Theta/dr \simeq 0$ as well as $|d\ln(n_0, T_0)/d\ln r| \gg 1$. The additional assumption $|d\ln(\Phi_0, U_{\|0})/d\ln r| \gg 1$ can only be justified a posteriori.

4. Numerical Evaluation and Discussion

Introducing the dimensionless variables $y_1^* = \Theta U_{\|0}/V_n$, $y_2^* = -E_{r0}/(V_n B_0)$, $2y_3^* = dy_1^*/d\rho$, and $y_4^* = dy_2^*/d\rho - y_3^*$ normalized to $V_n = -\,\kappa k T_{0,s}/(2eB_0 r_s)$, eqs.(3-5) pass into a set of two second order or equivalently four first order nonlinearly coupled ordinary differential equations. The latter have been numerically evaluated for the radial density and temperature profiles $n_0 = n_{0,s}\sqrt{1 + \tanh\xi}$; $T_0 = T_{0,s}(1 + \tanh\xi)$; $n_a = n_{a,s}\exp(-\xi)$ (n_a = atom density), where $\xi = \kappa(1 - \rho)$ and $\rho = r/r_s$ (the index s indicates the separatrix position). Besides $\Lambda_s^* = \kappa/(\Theta^2\Omega_0\tau_{0,s})$ and $\Lambda_s^{**} = R_0\sqrt{\nu_{ia0,s}\tau_{0,s}}/(\Theta v_{T0,s}\tau_{0,s})$ and the boundary conditions (prescribing $y_{1,2}^*$ at the core-edge interface and $y_{3,4}^*$ at the separatrix), the magnitude and the structure of the anomalous transport coefficients play a decisive role for the solution. In particular, their dependence on dE_{r0}/dr is important. Some of the models (the current diffusive ballooning model [6], for instance) predict $\eta_{2,0}^{an} = mn_0 D_{\perp 0}^{an}$ with $D_{\perp 0}^{an} \sim D_{\perp 0,s}^{an}/(1 + \alpha^* Z_I^2)$, where in the present case $Z_I = y_4^* + y_3^* - y_2^*/\rho$, although according to detailed fluctuation measurements on DIII-D [7] the above diffusion ansatz was only validated for negative radial electric field gradients. Therefore only preliminary results for nearly space independent anomalous

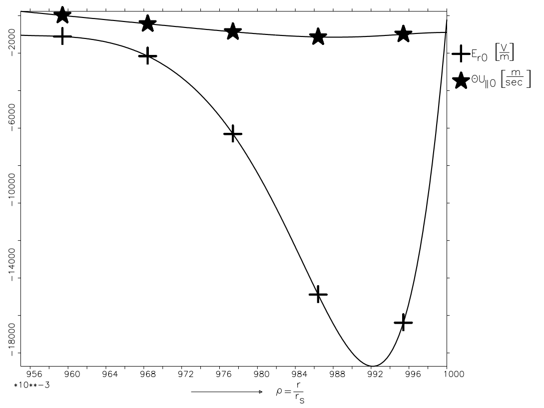

Figure 1: *E_{r0}[V/m] and $\Theta U_{||0}$[m/sec] inside the separatrix for normal $\vec{B}$ direction*

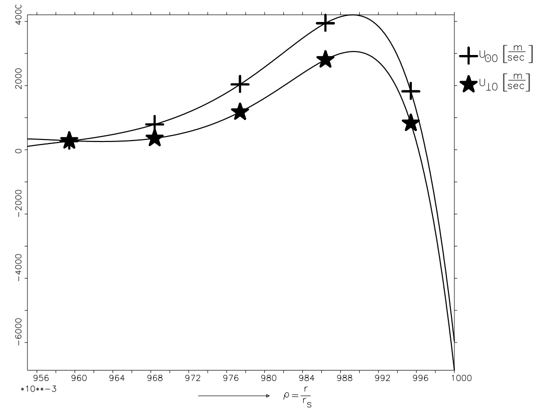

Figure 2: *$U_{\perp 0}$[m/sec] and $U_{\theta 0}$ [m/sec] inside the separatrix for normal $\vec{B}$ direction*

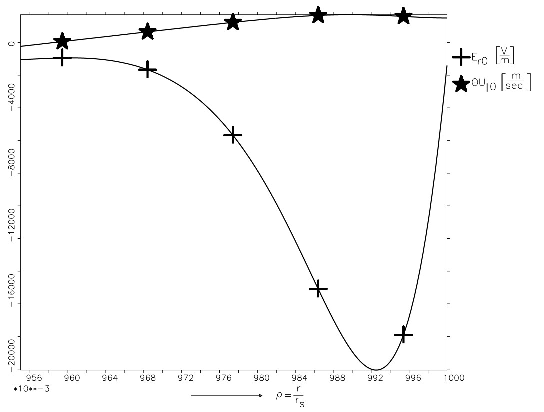

Figure 3: *E_{r0}[V/m] and $\Theta U_{||0}$[m/sec] inside the separatrix for inverted $\vec{B}$ direction*

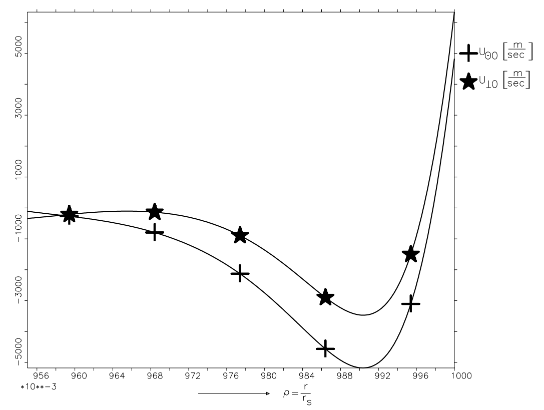

Figure 4: *$U_{\perp 0}$[m/sec] and $U_{\theta 0}$ [m/sec] inside the separatrix for inverted $\vec{B}$ direction*

diffusion and shear viscosity coefficients ($\alpha^* \ll 1$) are presented both for normal $\vec{B}$ (opposite to the induced toroidal current, Figs. 1,2) and for inverted $\vec{B}$ (Figs.3,4). E_{r0} and the ion rotation velocities are shown within a radial distance of $\simeq 2cm$ inside the separatrix assuming $D^{an}_{\perp 0} \simeq 0.3m^2/sec$, $\kappa = 60$, $n_{0,s} = 7 \times 10^{12}/cm^3$, $kT_{0,s} = 80eV$, $n_{a,s} = 10^{-3} n_{0,s}$, $u_{a,\phi} = 0\,cm/sec$ ($\rightarrow \Lambda^*_s \simeq 0.75$ and $\Lambda^{**}_s \simeq 0.28$). By decreasing $D^{an}_{\perp 0}$ or κ independently (although both should vary inversely) the E_{r0} profile steepens, its strong curvature zone being compressed towards the separatrix provided $dE_{r0}/dr|_s$ is increased to keep $E_{r0,s}$ constant. Simultaneously the poloidal ion rotation velocity passes a maximum/minimum within the narrow $\vec{E} \times \vec{B}$ shear layer just inside the separatrix (for normal/inverted $\vec{B}$ direction) thus indicating a decoupling from the standard nc profile.

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