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Cross Phase Evolution in Electrostatic Turbulence

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Abstract. Electrostatic turbulent transport of particles is a function of the amplitude, coherence, and cross phase of density and electrostatic potential fluctuations. The role of $\mathbf{E}\times\mathbf{B}$ flow shear in suppressing turbulent transport in H-mode and internal transport barriers is well known. Previous work on trapped electron mode turbulence has shown that flow shear can reduce the particle flux by phase modification while leaving amplitudes relatively unchanged. This feature, reduced particle flux with minimal change in fluctuation amplitudes, has been observed in experiments on DIII-D, JT-60U, and RFX. In this work, a model evolution equation for the cross phase between density and electrostatic potential fluctuations is derived. The evolution equation includes effects due to ExB flow shear and nonlinear phase modifications. The cross phase evolution equation has been incorporated into a predator-prey model for the transition dynamics of transport barriers. Implications for turbulence and transport barriers include lower thresholds for L-H transitions when cross phase evolution is included. This and other results will be discussed.

1. Introduction

Improved confinement regimes such as H-mode are believed to be a result of a reduction in turbulent transport [1]. Experimental measurements have shown reductions in fluctuation levels and decreases in turbulent correlation lengths during L-H transitions [2]. A number of experiments have observed that modification of cross phases also plays an important role in the reduction of turbulent transport during L-H transitions [3-6]. For example, the cross phase between density and electrostatic potential fluctuations, which controls the turbulent particle flux, has been shown to be strongly modified during L-H transitions on the DIII-D tokamak [4], on the RFX reversed field pinch [5], and even to change sign in measurements on the H-1 Heliac [6]. A similar change in sign was observed in the temperature - electrostatic potential cross phase in measurements on the TEXTOR tokamak [7].

Theoretical work on of the effect of $\mathbf{E}\times\mathbf{B}$ flow shear on fluctuations initially examined reductions in fluctuation levels and correlation lengths [8]. This work led to the development of phase transition models which described the dynamics of L-H transitions [9]. Concurrently work on resistive pressure gradient driven turbulence (RPGDT) showed that cross phases are also modified in the presence of $\mathbf{E}\times\mathbf{B}$ flow shear [10, 11, 12].

In this paper, a mean field evolution equation for the cross phase is derived. In section 2, both a general equation, which is independent of the turbulence model used, and an equation specific to two-field RPGDT are derived. In section 3, a cross phase evolution equation is incorporated into a phase transition model of the L-H transition. Cross phase effects are shown to modify transition dynamics and thresholds. Finally, section 4 contains a brief discussion.

2. Cross Phase Evolution Equation

The cross phase, $\delta_{\mathbf{k}}$, between the Fourier transform of density and radial ExB velocity fluctuations, $n_{\mathbf{k}}$ and $v_{\mathbf{k}}$, is defined in the equation $\langle n_{\mathbf{k}} v_{-\mathbf{k}} \rangle = \langle n_{\mathbf{k}} \rangle_{rms} \langle v_{\mathbf{k}} \rangle_{rms} e^{i\delta_{\mathbf{k}}}$. This relation assumes that $n_{\mathbf{k}}$ and $v_{\mathbf{k}}$ are fully coherent. Hence, solving for the cross phase as

$$\delta_{\mathbf{k}} = \frac{i}{2} \ln\left[\frac{\langle n_{\mathbf{k}} v_{-\mathbf{k}} \rangle}{\langle n_{-\mathbf{k}} v_{\mathbf{k}} \rangle} \right] \tag{1}$$

yields an equation which is independent of the coherence. Note that this cross phase is identical to that between density and the fluctuating radial electric field. Taking a time derivative yields a cross phase evolution equation

$$\frac{d}{dt}\delta_{\mathbf{k}} = \frac{i}{2}\left[\frac{1}{\langle n_{-\mathbf{k}} v_{\mathbf{k}} \rangle} \frac{d}{dt}\langle n_{-\mathbf{k}} v_{\mathbf{k}} \rangle - \frac{1}{\langle n_{\mathbf{k}} v_{-\mathbf{k}} \rangle} \frac{d}{dt}\langle n_{\mathbf{k}} v_{-\mathbf{k}} \rangle \right]. \tag{3}$$

In order to derive a mean field equation for the cross phase, a two field model will be used for the density and radial velocity fluctuations, namely, $dn_{\mathbf{k}} / dt = L_{\mathbf{k}}^{nn} n_{\mathbf{k}} + L_{\mathbf{k}}^{nv} v_{\mathbf{k}}$ and $dv_{\mathbf{k}} / dt = L_{\mathbf{k}}^{vv} v_{\mathbf{k}} + L_{\mathbf{k}}^{vn} n_{\mathbf{k}}$ where the terms $L_{\mathbf{k}}^{nn}$, $L_{\mathbf{k}}^{nn}$, $L_{\mathbf{k}}^{nn}$, and $L_{\mathbf{k}}^{nn}$, are all operators from a renormalization of the basic model. The form of the operators will depend on the model used. We will leave these in this general form for the moment but later, as a more specific case for pressure gradient driven turbulence [9], these are $L_{\mathbf{k}}^{nn} = i\Omega - k_y^2 W^2 (dv_0 / dr)^2 / 4D - D / 2W^2$, $L_{\mathbf{k}}^{nv} = -dn_0 / dr$, $L_{\mathbf{k}}^{vn} = \kappa k_y^2 W^2 / \rho_m$, and $L_{\mathbf{k}}^{nn} = -k_y^2 W^2 (dv_0 / dr)^2 / 4D - \mu / 2W^2 - \gamma_0 W^4 / W_0^4$, where the symbols are defined in Ref. 10. These terms have been averaged over a mode width assuming a shifted Gaussian, $\exp\left[-(x - i\xi)^2 / (2W)^2 \right]$ with the shift due to flow shear, $\xi = k_y W^3 (dv_0 / dr) / D$ [9]. The flow shear, dv_0 / dr, is assumed constant over the mode width. This precludes cases such as Boedo et al. have shown where the variation of the shear is on the scale of a mode width [6] in which case it may be controlling the mode structure.

Using the general form of the operators in Eqn. (3), we obtain the cross phase evolution equation

$$\begin{aligned} \frac{d}{dt}\delta_{\mathbf{k}} = &\left\{ \mathrm{Im}\left[L_{\mathbf{k}}^{nv}\right]\alpha - \mathrm{Im}\left[L_{\mathbf{k}}^{vn}\right]\alpha^{-1} \right\} \cos\delta_{\mathbf{k}} - \left\{ \mathrm{Re}\left[L_{\mathbf{k}}^{nv}\right]\alpha + \mathrm{Re}\left[L_{\mathbf{k}}^{vn}\right]\alpha^{-1} \right\} \sin\delta_{\mathbf{k}} \\ &+ \left\{ \mathrm{Im}\left[L_{\mathbf{k}}^{nn}\right] - \mathrm{Im}\left[L_{\mathbf{k}}^{vv}\right] \right\} \end{aligned} \tag{3}$$

where we have introduced the equipartition ratio, $\alpha \equiv \langle v_{\mathbf{k}} \rangle_{rms} / \langle n_{\mathbf{k}} \rangle_{rms}$. This general form of the equation is independent of the turbulence model used.

For RPGDT turbulence, the equation simplifies to

$$\frac{d}{dt}\delta_{\mathbf{k}} = \left\{ \alpha \frac{dn_0}{dr} - \alpha^{-1} \frac{\kappa k_y^2 W^2}{\rho_m} \right\} \sin\delta_{\mathbf{k}} - \{\Omega_{\mathbf{k}}'\} \cos\delta_{\mathbf{k}} \tag{6}$$

where the nonlinear phase shift has been assumed to depend on the cross phase as $\Omega_{\mathbf{k}} = \Omega_{\mathbf{k}}' \cos\delta_{\mathbf{k}}$ since nonlinear frequency shifts are sensitive to the cross phase between density and potential. The dependence of the density gradient on $\sin\delta_{\mathbf{k}}$ makes physical sense in the following manner. Typically $dn_0 / dr < 0$, and thus $(dn_0 / dr)\sin\delta_{\mathbf{k}} < 0$ for $\delta_{\mathbf{k}} > 0$. This term pushes the cross phase towards 0 which yields maximum transport down the density gradient.

3. Phase Transition Model with Cross Phase Evolution

Phase transition models have been successful at describing the basic dynamics of L-H transitions [9]. In this section, cross phase dynamics are included in a phase transition model in order to study the effects of cross phase evolution on transition dynamics. The details of the model without cross phase effects (i.e., a constant cross phase is assumed) are given in Ref. 10. Here we will focus on the additional terms arising from cross phase evolution. The full model couples the fluctuation amplitude, E, the pressure gradient, N, the radial derivative of the $\mathbf{E}\times\mathbf{B}$ shear flow, V, the poloidal flow, U, and the cross phase, δ. The full model in normalized variables is

$$\frac{\partial E}{\partial \tau} = EN - k_1 E^2 \left(1 + k_5 \cos\delta\right) - EV^2 \tag{7}$$

$$\frac{\partial U}{\partial \tau} = k_2 EV\left(1 + k_6 \cos\delta\right) - k_3 U \tag{8}$$

$$\frac{\partial N}{\partial \tau} = -EN \cos\delta - N + Q \tag{9}$$

$$U = V - k_4 N^2 \tag{10}$$

$$\frac{\partial \delta}{\partial \tau} = -k_7 N \sin\delta - k_8 E \sin\delta + k_9 E \cos\delta + k_{10} V^2 \cos\delta \tag{11}$$

where Q is the power input, and the k's are physical parameters. The time unit, τ, is normalized to a neoclassical diffusion time. The cross phase appears a number of times in the model: in the fluctuation equation (7), nonlinear energy transfer has been assumed to depend on the cross phase (as is the case for the $\mathbf{E}\times\mathbf{B}$ nonlinearity); in the poloidal flow equation (8), the Reynolds stress depends indirectly on the cross phase (analogous to a convective heat flux, momentum is carried with the particle flux); the anomalous particle flux in Eqn. (9) is directly dependent on the cross phase; and finally, it appears in the cross phase evolution equation (11).

The model has three nontrivial fixed points: (i) a low power fixed point with fluctuation amplitude scaling linearly with power input and zero poloidal flow; (ii) a medium power fixed point with fluctuation amplitude insensitive to power input (above the threshold) and a poloidal flow; and (iii) a high power fixed point where the fluctuations are completely quenched and the $\mathbf{E}\times\mathbf{B}$ flow shear is due solely to the pressure gradient. For simplicity, we will call these fixed points (i) "L-mode", (ii) "H-mode", and (iii) "quenched H-mode". These are the same fixed points as without cross phase evolution but cross phase effects to alter the thresholds for transition and the dynamics. The behavior of the model is complex and in the following, an example of how the cross phase affects the transition from one fixed point to another is given.

Without cross phase effects, $\overline{Q}_{critical} = k_1 k_3\left(k_3 + k_2\right)/\, k_2^{\,2}$ is the critical power for transition from L-H. In Figure 1, plots of the fluctuation amplitude, pressure gradient, and $\mathbf{E}\times\mathbf{B}$ flow shear for a case of subcritical power input (a) without and (b) with cross phase effects. While the cross phase effects clearly lower the power threshold for a transition, the effect is complicated. The control of the particle flux and the enhancement of the Reynolds stress both act to lower the threshold while the additional nonlinear transfer acts to raise the power threshold.

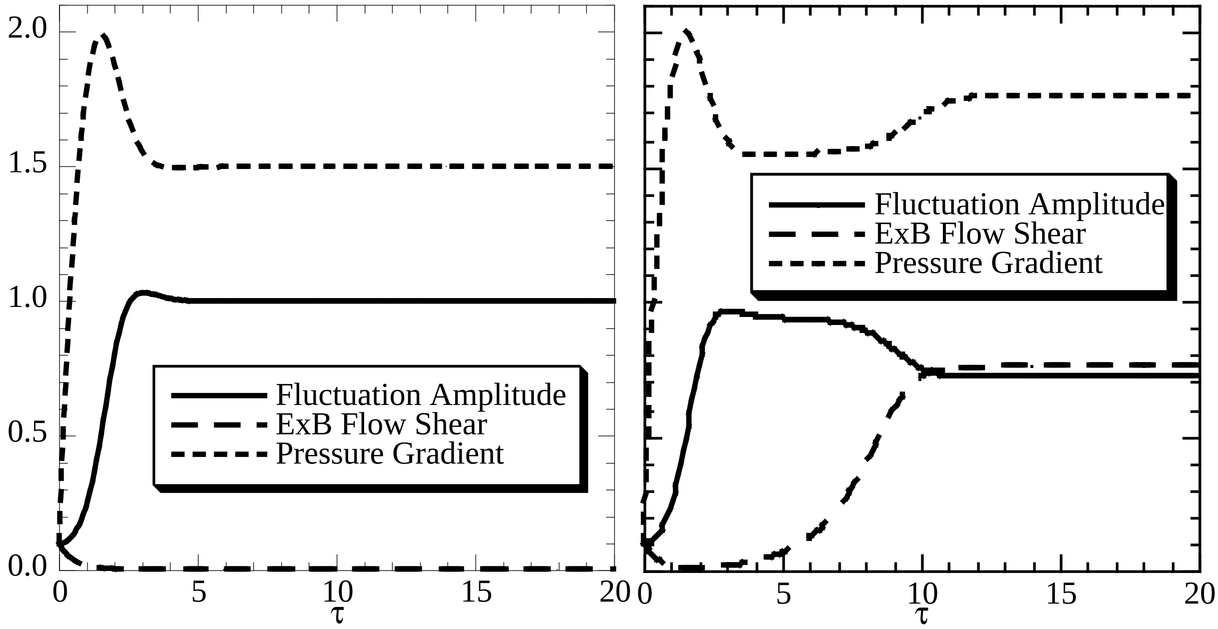

Figure 1 Evolution of E, N, and V for cases run (a) without and (b) with cross phase effects. The parameters used were $Q = 7$, $k_1 = 1.5$, $k_2 = k_3 = 2$, $k_4 = 0$, $k_5 = 0.1$, $k_6 = 0.4$, and $k_7 = k_8 = 2k_9 = k_{10} = 1$.

4. Discussion

A cross phase evolution equation for RPGDT has been developed. We have shown that cross phase effects included in a phase transition model quantitatively but not qualitatively modify the power threshold for transitions from L-H mode. A longer paper to follow this will clarify the various cross phase effects in the phase transition model.

One of the limitations of this theory is its assumption of weak flow shear. Future work will address that limitation.

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References

[1] Carreras B A 1997 *IEEE Plasma Sci.* **25** 1281
[2] Burrel K 1997 *Phy. Plasmas* **4** 1499
[3] Tynan G R *et al* 1994 *Phys. Plasmas* **1** 3301
[4] Moyer R A *et al* 1999 *Plasma Phys. Control. Fusion* **41** 243
[5] Antoni V *et al* 2000 *Plasma Phys. Control. Fusion* **42** 83
[6] Shats M and Rudakov D L 1997 *Phys. Rev. Lett.* **79** 2690
[7] Boedo J *et al* 2000 *Phys. Rev. Lett.* **84** 2630
[8] Biglari H. Diamond P H, and Terry P W 1990 *Phys. Fluids B* **2** 1
[9] Diamond P H *et al* 1994 *Phys. Rev. Lett.* **72** 2565
[10] Carreras B A *et al* 1995 *Phys. Plasmas* **2** 2744
[11] Ware A S *et al* 1996 *Plasma Phys. Control. Fusion* **38** 1343
[12] Ware A S, Terry P W, Carreras B A and Diamond P H 1998 *Phys. Plasmas* **5**, 173