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Evaluation of the parallel current density in a stellarator using the integration technique along the magnetic field line*

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Introduction

A method based on an integration procedure along the magnetic field lines for calculating the bootstrap current in stellarators had been proposed in [1]. In that work, the contribution of trapped particles to the bootstrap current had been neglected because it is small if the modulation of the magnetic field module along the magnetic field line is small. Also, Boozer coordinates for representing the magnetic field had been used and results for the magnetic surface averaged value of the parallel current density had been obtained. In the present paper the procedure of [1] is generalized in order to take into account also the contribution of trapped particles to the equilibrium current. This is achieved in analogy to the results of [2] where a solution of the drift-kinetic equation that takes into account all possible classes of trapped particles had been obtained in order to derive $1/\nu$ transport coefficients. In the present work, an expression for local parallel current density is derived in real-space coordinates. Using this expression and the earlier developed method for the calculation of the normal to the magnetic surface, a technique is developed to calculate the local and the surface averaged values of parallel equilibrium current density directly in real-space coordinates.

Basic formula and parameters

As in [1], the starting point is the linearized drift-kinetic equation in the long-mean-free-path regime with a simplified Lorenz collision operator which describes pitch angle scattering but does not conserve momentum,

$$\sigma \frac{\partial \tilde{f}}{\partial s} + \frac{V^\psi}{|v_{||}|} \frac{\partial f_M}{\partial \psi} = 4\nu A \frac{\partial}{\partial J_\perp} \left(\frac{|v_{||}| J_\perp}{B} \frac{\partial \tilde{f}}{\partial J_\perp} \right), \tag{1}$$

where σ is the sign of parallel velocity, s is the distance measured along the magnetic field line, ψ is the magnetic surface label, $V^\psi = \mathbf{V} \cdot \nabla\psi$ is a radial component of the drift velocity, $v_{||}^2 = v^2 - J_\perp B$, $J_\perp = v_\perp^2/B$ is the perpendicular adiabatic invariant, B is the magnetic field module, f_M is the Maxwellian distribution function, and νA is the pitch-angle scattering frequency. As discussed in [1], for small magnetic field modulations within the magnetic surface, the momentum preserving term will change the resulting value of the average parallel current by a factor which is weakly dependent on the magnetic field geometry and, therefore, can be taken from tokamak theory. The solution to (1) is looked for in a series expansion with respect to the collision frequency,

$$\tilde{f} = f_{-1} + g_0 + f_0 + g_1 + f_1 + \ldots, \tag{2}$$

where $f_k, g_k \sim \nu^k$ and f_k is constant whereas g_k varies along the magnetic field line. In [2] we obtained the leading order term in this expansion f_{-1} taking into account all possible classes

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of trapped particles. The leading term is enough to obtain the particle and energy fluxes in the $1/\nu$ regime. In the present work we also derive g_0 and f_0 which are necessary for the computation of the parallel current density. Different to $1/\nu$ transport, where the contribution of multiply trapped particles within many local magnetic field minima is small, they play an essential role in the formation of the parallel current density. The problem in the interpretation of the boundary condition at the trapped-passing boundary in [1] where the "last" class of trapped particles cannot be identified, does not appear if instead of an irrational surface one first considers a rational surface. In this case, the number of classes of trapped particles stays finite, and the boundary conditions are clearly defined. Then, the irrational surface can be considered as a limit case of a "true" rational surface [3] which satisfies the closure condition for the equilibrium currents (Pfirsch-Schlüter),

$$Y_{PS}(L) = 0, \qquad Y_{PS}(s) = \int\limits_0^s \mathrm{d}s' \frac{|\nabla\psi| k_G}{B^2}, \tag{3}$$

where L is the full magnetic field period and $k_G = (\mathbf{h} \times (\mathbf{h} \cdot \nabla)\mathbf{h}) \cdot \nabla\psi / |\nabla\psi|$ is the geodesic curvature of the magnetic field line. When (3) is satisfied, the expression for the local parallel current density is convergent and has the following form,

$$\frac{j_{||}}{B} = -c\,\lambda_{||}\,\frac{1}{B_0^2}\frac{dp}{dr}, \qquad \lambda_{||} = \lambda_{PS}(s) + \lambda_B, \tag{4}$$

where c is the speed of light, p is the plasma pressure, and B_0 is some reference magnetic field. The radial derivative of the pressure and the magnetic surface averages of any function A of spatial coordinates are given by

$$\frac{dp}{dr} \equiv \frac{dp}{d\psi} < |\nabla\psi| >, \qquad < A >= \lim_{L\to\infty} \left(\int\limits_0^L \frac{\mathrm{d}s}{B} \right)^{-1} \int\limits_0^L \mathrm{d}s \frac{A}{B}. \tag{5}$$

The dimensionless quantities λ_{PS} and λ_B in (4) which characterize the magnetic field geometry are

$$\lambda_{PS}(s) = \frac{2B_0^2}{< |\nabla\psi| >} \left[Y_{PS}(s) - Y_{PS}(s_m) \right], \tag{6}$$

$$\lambda_B = \frac{3B_0^2}{8 < |\nabla\psi| >} \lim_{L\to\infty} \frac{1}{v^3} \int\limits_0^{J_{\perp min}^{(abs)}} \mathrm{d}J_\perp J_\perp^2 \left[\frac{1}{I_L} \int\limits_0^L \mathrm{d}s \frac{|v_{||}|}{B} Y_B(s) - Y_B(s_m) \right], \tag{7}$$

$$Y_B(s) = \int\limits_0^s \mathrm{d}s' \frac{B|\nabla\psi| k_G}{|v_{||}|^3}, \tag{8}$$

$$I_L = \int\limits_0^L \mathrm{d}s \frac{|v_{||}|}{B}, \tag{9}$$

where $J_{\perp min}^{(abs)} = v^2/B_{max}^{abs}$ corresponds to the trapped-passing boundary, B_{max}^{abs} is the global maximum of B on the particular magnetic field line, and s_m is the position of this maximum. For L being big enough, B_{max}^{abs} approaches the global maximum of the magnetic field on the surface. The integration along the magnetic field line must be performed simultaneously with the $\nabla\psi$ calculation actually done by solving additional linear differential equations when integrating along the field line (see, e.g., [2]).

Contrary to λ_B, the quantity λ_{PS} is a function of s. One can show that the varying part of λ_{PS} which corresponds to the Pfirsch-Schlüter current, is the same as it is obtained from ideal MHD equilibrium equations. However, those equations do not restrict the constant part of the parallel current. The missing constant part of $j_{||}/B$ is given by an average value of $j_{||}/B$. Two definitions of this average are commonly used. The first one, more suitable for equilibrium studies, corresponds to the toroidal current density averaged over the area between two close magnetic surfaces. It is obtained from (4) if $\lambda_{||}$ is replaced with λ_{b1},

$$\lambda_{b1} = \frac{\langle \lambda_{||} B^{\phi} \rangle}{\langle B^{\phi} \rangle} = \frac{\langle \lambda_{PS} B^{\phi} \rangle}{\langle B^{\phi} \rangle} + \lambda_B, \tag{10}$$

where $B^{\varphi} = \mathbf{B} \cdot \nabla\varphi$ is the toroidal contravariant component of the magnetic field and with surface averages defined in (5). The second definition used in [1] corresponds to the case when the average parallel current vanishes completely in the Pfirsch-Schlüter regime,

$$\lambda_{b2} = \frac{\langle \lambda_{||} B^2 \rangle}{\langle B^2 \rangle} = \frac{\langle \lambda_{PS} B^2 \rangle}{\langle B^2 \rangle} + \lambda_B. \tag{11}$$

For comparison, one can transform the expression for the bootstrap current derived in magnetic coordinates in [1] to real-space coordinates. The resulting expression would yield instead of λ_{b2} the quantity λ_{bB}. The difference between the two results, $\delta\lambda_b = \lambda_{bB} - \lambda_{b2}$ is given as

$$\delta\lambda_b = \frac{2B_0^2}{< |\nabla\psi| >} \left[\frac{\langle Y_\delta(s) B^2 \rangle}{\langle B^2 \rangle} - Y_\delta(s_m) \right], \tag{12}$$

$$Y_\delta(s) = \int\limits_0^s \mathrm{d}s' \frac{|\nabla\psi| k_G}{B^2} \left(1 - \frac{B}{B_{max}^{abs}} \right)^{3/2}. \tag{13}$$

In accordance with the estimate performed in [1], this quantity is small compared to λ_{b2} if the modulation amplitude of the magnetic field module within the magnetic surface is small.

Results

The proposed technique has been applied to two magnetic configurations, a simplified $l = 3$ configuration with parameters of the Uragan-3M torsatron (U-3M) and the vacuum W-7X configuration. For the magnetic field, the representation in real space coordinates has been used (see [4] for details). To simplify the comparison, we introduce the normalized quantity $\hat{\lambda}_{b1} = \lambda_{b1} \iota \sqrt{r/R}$ where ι is the rotational transform angle in 2π units, r is an average radius and R is the big radius of the torus. This quantity is unity for a tokamak with a large aspect ratio. The results of the calculation are shown in Fig. 1. The simplified U-3M configuration represents a standard stellarator with both the helical modulation and rotational transform rapidly decreasing towards the magnetic axis. The bootstrap current for this configuration is close to that of a tokamak as can be seen from the behavior of $\hat{\lambda}_{b1}$ which is positive and is close to unity in a major part of the considered region. At the same time, the W-7X configuration has been optimized in order to reduce the bootstrap current. For this configuration $|\hat{\lambda}_{b1}|$ does not exceed 0.25 and at the outer region this quantity changes sign from negative to positive values.

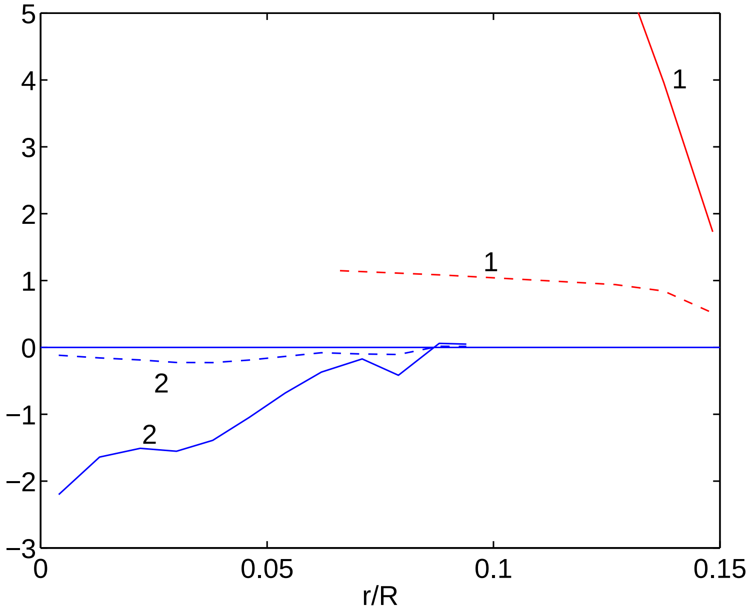

Fig. 1. Parameters λ_{b1} (solid) and $\hat{\lambda}_{b1}$ (dashed) for Uragan-3M (label 1) and W-7X (label 2) as a functions of the aspect ratio r/R.

Summary

A technique for calculating the parallel equilibrium plasma current with the magnetic configurations given in real space coordinates has been developed. Basically, the method is similar to the method proposed in [1] where the magnetic configuration is given in Boozer coordinates. In addition, the contribution of trapped particles which had been neglected in [1] is recovered in the present work. As a result, the local current density calculated in this way is shown to be consistent with the results obtained from ideal MHD equilibrium equations.

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