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The Hurst exponent and long-time correlation

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The nature of turbulent cross-field transport is still a problem in discussion in magnetically confined plasmas. In particular, in recent years, some models based on an avalanche type of transport have been proposed to the community. These models are based on an auto organisation process and on the existence of critical gradients in the plasma, which are producing the avalanches. Thus, it is suspected that the self-organised-criticality (SOC) [1] mechanism may play an important role in the cross-field turbulent transport. The models based on this approach predict some characteristics of the experimental signal such as the existence of very long time correlations. Self-similar random processes were introduced by Mandelbrot [2-3] to model this long run behaviour. A rescaled range statistics (R/S) method was proposed [4] to evaluate the Hurst exponent (*H*) in order to determine these long-time dependencies. It is shown that when $1 > H > 0.5$, there is persistence and when $0.5 > H > 0$, there is anti-persistence. In several devices [5], H has been calculated to range from 0.62 to 0.72, which is assumed to indicate the existence of an algebraic decay of the autocorrelation function for long time lags, i.e., the existence of long-time correlations in the fluctuations of the ion saturation current at plasma edge. The result is supposed to be compatible with transport by avalanches.

In the present study, we apply the R/S method to fluctuations of the ion saturation current measured by a Langmuir probe at the edge of Tore Supra. It is observed that *H* is well above 0.5 in the long time-range. However, it is found that the information which leads to $H > 0.5$ is totally contained in the short-time correlation and no link to long times is found.

The R/S statistics

The rescaled range statistics (R/S) method was proposed [4] to evaluate the Hurst exponent (*H*) to determine long-time dependencies in various signals. For a time series of length *n*, $X = \{X_t : t = 1,2,\cdots,n\}$, the *R/S* ratio is defined as the ratio of the maximal range of the integrated signal normalised to the standard deviation:

$$\frac{R(n)}{S(n)} = \frac{\max(0,W_1,W_2,\cdots,W_n) - \min(0,W_1,W_2,\cdots,W_n)}{\sqrt{S^2(n)}}$$

Here $W_k = X_1 + X_2 + \cdots + X_k - k\overline{X}(n)$, where $\overline{X}(n)$ and $S^2(n)$ are respectively the mean and variance of the signal. The expected value of *R/S* scales like cn^H as $n \to \infty$, where *H* is called Hurst exponent. Hurst *et al* have found [4] that $H = 0.5$ for random data, and $H > 0.5$ for biased random data. Detecting a constant slope in a certain range of the R/S curve (in the loglog plot) may be a signature of the self-similarity of the signal in the same range.

The accuracy in the determination of H depends at all time lags on the number of data points which are used for the calculation. Provided that this number of points is reasonably high, (i.e., several times the maximum time lag which is plotted), the R/S curve is expected to give information on self-similarity at all time lags

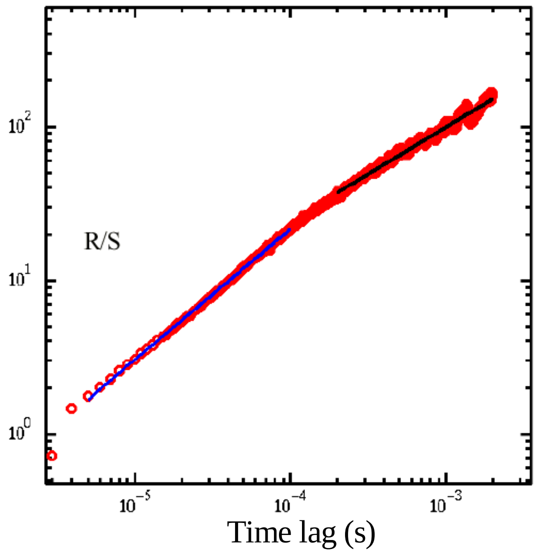

Figure 1: *(a) R/S curve calculated on the ion saturation current of a movable probe (shot 25164). Two constant slopes can be clearly seen on the graph. The first one corresponding to the frequency spectrum gives H = 0.85. The second one which extends from 200 µs up to 2 ms gives H = 0.61.*

For example, Fig. 1 shows the R/S curve calculated directly on the ion saturation current from a limiter shot with the probe at r/a=0.99. This curve clearly displays two regions where the slope is constant. The first one corresponds to the time region where the frequency spectrum has a constant slope and extends from 5 to 100 µs. The second one corresponds to the lower part of the frequency spectrum (below 10 kHz) where in general the statistics is poor because of limited data length (8000 points). The basic idea for detecting long time correlations is to use the analysis of the R/S curve at large time lags well above the autocorrelation time (typically ten times).

Shuffling Method

In order to test the effect of correlations at all time scales on the determination of H, we use a shuffling procedure which allows us to destroy selectively the correlations in the signal. The data is divided into blocks of length ΔN and the blocks are shuffled randomly. By using this method, we destroy any correlations existing at time scales larger than ΔN. As the shuffling may still allow some correlations to exist, we repeat it ten times and take the average over the ten H values. To test the effect of presence or absence of correlations at all time scales, the size of the blocks is varied continuously from ΔN = 1 (complete randomisation) to ΔN = length of the data (no shuffling). To test the long range correlations on our signals, we will restrict the R/S curve to the maximum time lag of 1 ms, because our data length corresponds to 8 ms. We will look at the slope of the R/S curve, i.e., we will measure the self-similarity in the range 200 µs to 1 ms. It is clear from Fig. 1 that the slope of R/S is constant in this range and that the number of points used for this analysis is much larger than the largest lag time, so that a priori all the conditions to perform the analysis are fulfilled. Fig. 2 presents the result of the shuffling procedure applied on the data from a fixed probe in the Scrape Off layer of Tore Supra. Fig. 2a shows the value of the H parameter as a function of the block length, and Fig. 2b shows the autocorrelation function of the signal. One can see that the H parameter determined in the range 200-1000 µs changes for blocks with $\Delta N < t_0$ and remains unchanged for blocks with $\Delta N > t_0$. This means that the H parameter calculated in the 200-1000 µs range is determined by the correlations existing for $t < t_0$, i.e., by the correlations at short times [6].

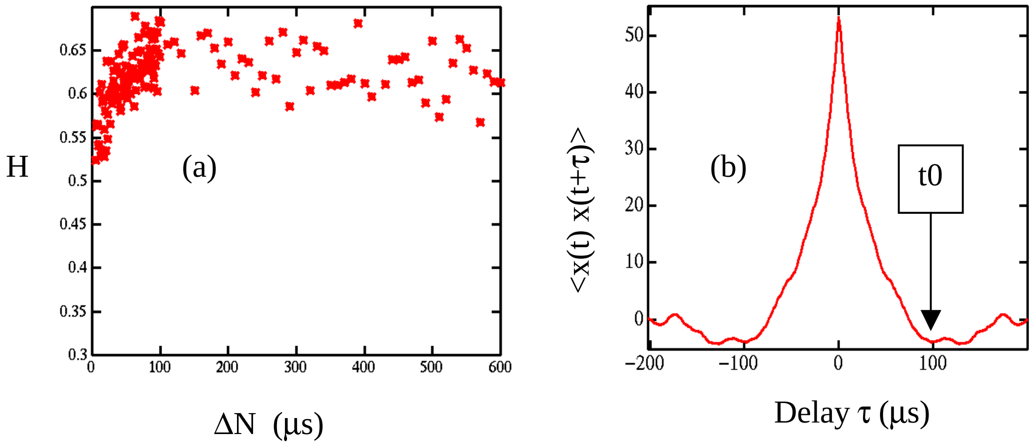

Figure 2: *(a) H exponent calculated directly on the probe signal in the 200-1000 µs range on the R/S curve as a function of the block size used for the shuffling procedure. A block of length ΔN only allows correlations at times < ΔN to remain in the signal once the shuffle is performed. (b) autocorrelation function of the probe signal. The main point in this figure is that the H parameter remains constant for blocks of the length of the autocorrelation function.*

R/S versus Spectral analysis

One of the properties of self-similar random processes, i.e., Fractional Brownian Motions [3], is the relationship between the H exponent and the characteristic exponent γ of the $f^{-\gamma}$ power law frequency spectrum typical for these signals. It is found that :

$$H = (\gamma - 1)/2 \qquad (1)$$

It must be stressed that this relation implies that H is determined from an R/S analysis performed on the increments of the signal (i.e. the derivative of the signal). To be more specific, considering for example a classical Brownian motion with, i.e., fully uncorrelated increments making a white noise, the R/S analysis performed on such increments gives H = 0.5.

For any Fractional Brownian Motion the correlation between past and future increments is the same at all time and depends only on the H parameter:

$$C(t) = \langle (B_H(0) - B_H(-t))(B_H(t) - B_H(0)) \rangle / \langle B_H(t)^2 \rangle = 2^{2H-1} - 1 \qquad (2)$$

Directly related to this property, i.e., persistence or antipersistence, are an algebraic decay of the autocorrelation function and the $f^{-\gamma}$ power law. The same power law extends over all frequencies and thus a constant slope is observed in the R/S plot. In the case of the ion saturation current several time, resp. frequency, domains exist as shown above. Performing the R/S analysis on the time derivative of the Langmuir probe data gives different slopes clearly associated to these different time domains. In Fig. 3a and Fig. 3b the R/S curve and the spectral density are plotted, respectively. As expected, the H values are in agreement with the slopes measured from the loglog plot of the frequency spectrum. The value H = 0.57 in the lags-range 10-200 µs reflects the self-similar behaviour of the spectral density in the high frequency domain. In the low frequency range the signal has a nearly flat spectrum, i.e., behaves as the spectrum of increments of classical Brownian motion (fractional Brownian noise). Nevertheless in the long time range the R/S statistics keeps memory of the self-similarity behaviour at short time lags. The H values in the range 0.6-0.7 observed in many devices [5] at large time lags and obtained by performing the R/S analysis directly on the signal reflect this behaviour. Moreover we found that the properties of the ion saturation current fluctuations compare very well with the ones of a modified fractional Brownian motion exhibiting a similar spectral density profile.

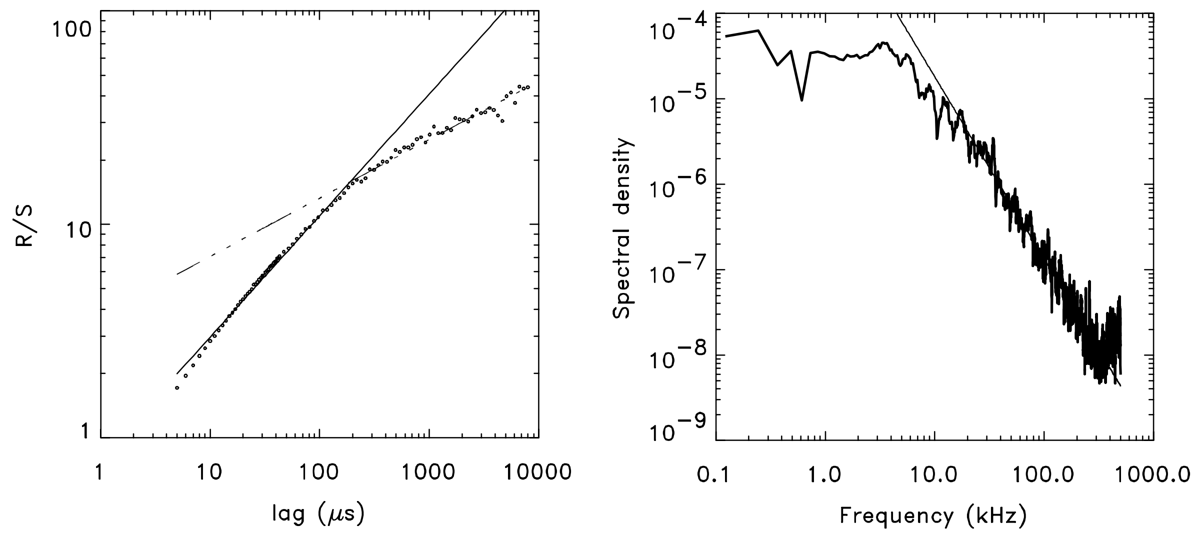

Figure 3: *(a) R/S curve calculated on the increments of the probe signal (shot 22253), the slope in the lags-range 10-200 μs and 200-4000 μs are H=0.57 and H=0.27 respectively. (b) frequency power spectrum of probe signal 22253, the slope from 10 to 300 kHz is* $\gamma=2.13$.

Conclusion

The R/S statistics calculated directly on the ion saturation currents of a probe in Tore Supra gives an H value well above 0.5 in a range which extends from 200 μs up to 2 ms. However, this value is found to be determined by the correlations of the signal at very short times. We conclude that an H value greater than 0.5 calculated on the signal at large time lags does not necessarily imply any persistence property at these long time scales because the values of $H >$ 0.5 obtained at large time lags are determined by the correlations at short times. The reason for this apparent contradiction may come from an incorrect interpretation of the R/S statistics applied on the signal and not on its increments (time derivative of the signal), the latter being the standard procedure to study fractional Brownian motions. As a consequence, one must be cautious when analysing the R/S curves calculated directly on the signal.

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