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FAST MAGNETOSONIC WAVE TRANSMISSION THROUGH CYCLOTRON RESONANCE LAYER UNDER PROPAGATION ACROSS NONUNIFORM MAGNETIC FIELD

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Abstract. Within the framework of the perturbation theory there is considered the propagation of fast magnetosonic waves through the cyclotron resonance layer across the magnetic field with straight lines of force in small β plasma. The transmission, reflection and absorption coefficients are found.

Introduction. The absorption of fast magnetosonic waves (FMSW) propagating across the magnetic field in small β plasma has been determined in Ref. [1] for the resonance at the second harmonic of the ion cyclotron frequency and in Ref. [2] for the arbitrary multiple resonances. The treatment of Refs [1,2] is based on employing the local dispersion equation valid for a locally uniform plasma whose properties experience little variation on the distance of the order of the wavelength. In the case considered the width of the cyclotron resonance region has the order of the Larmor radius of particles possessing thermal velocities, this radius being considerably less than the FMSW wavelength. Therefore it is necessary to justify the validity of the results reported in [1,2]. The presence of such a resonance layer leads not only to the FMSW absorption but also to their reflection from this layer. The present report deals with the FMSW propagation through the narrow layer of the cyclotron resonance in small pressure plasma across the magnetic field with straight lines of force within the framework of perturbation theory without using the WKB approach. The absorption, reflection and transmission coefficients are found.

1. Dielectric permittivity tensor. FMSW propagation across the magnetic field in the direction of nonuniformity (the X-axis) is determined by Maxwell's equations. From them there follow two integro-differential equations:

$$\partial^2 E_y / \partial x^2 = -\omega^2 / c^2 \left[\hat{\varepsilon}_{21}(x) E_x(x) + \hat{\varepsilon}_{22}(x) E_y(x)\right], \tag{1}$$

$$\hat{\varepsilon}_{11}(x) E_x(x) + \hat{\varepsilon}_{12}(x) E_y(x) = 0 . \tag{2}$$

Components of FMSW electric and magnetic fields $\boldsymbol{E}=(E_x,E_y,0)$ and $\boldsymbol{B}=(0,0,B_z)$ are proportional to $exp(-i\omega t)$. Components of the tensor $\hat{\varepsilon}_{ij}(x)$ are equal to

$$\begin{aligned} \hat{\varepsilon}_{11}(x) &= \varepsilon_1 + \delta\varepsilon_{11}(x) , \quad \hat{\varepsilon}_{22}(x) = \varepsilon_1 + \delta\varepsilon_{22}(x), \\ \hat{\varepsilon}_{12}(x) &= i(\varepsilon_2 + \delta\varepsilon_{11}(x)), \quad \hat{\varepsilon}_{21}(x) = -i(\varepsilon_2 + \delta\varepsilon_{22}(x)). \end{aligned} \tag{3}$$

Here

$$\varepsilon_1 = 1 - \sum_i \omega_{pi}^2 / \left(\omega^2 - \omega_{ci}^2\right) , \qquad \varepsilon_2 = -i \sum_i \omega_{pi}^2 \omega / \left(\omega^2 - \omega_{ci}^2\right) \omega_{ci} . \tag{4}$$

The operator $\left|\delta\hat{\varepsilon}_{ij}(x)\right|$ determines the electric current density of resonance particles

$$\delta\hat{\varepsilon}(x)\mathbf{E}(x) = \mathbf{j}_{res}(x) 4\pi i/\omega \tag{5}$$

$$\mathbf{j}_{res}(x) = \iint e_\alpha \mathbf{v} \tilde{f}(x, \mathbf{v}, k_x) e^{ik_x x} dk_x d\mathbf{v} \tag{6}$$

Here $\tilde{f}(x, \mathbf{v}, k_x)$ -is the perturbed distribution function of resonance ions due to the Fourier-component of the FMSW field ~ exp($ik_x x$). Expression for $\tilde{f}$ can be found from the linearized Vlasov equation through integrating along unperturbed trajectories.

$$\tilde{f}(x,\mathbf{v},k_x)=\frac{ie_\alpha n_{0\alpha}}{m_\alpha(2\pi)^{5/2}v_{T\alpha}^5}e^{-v^2/2v_{T\alpha}^2}\left(\omega-n\omega_{c\alpha}(x)-nv_y/L_B\right)^{-1}\times$$
$$\times\left\{J_n(\xi)E_x(k_x)n\omega_{c\alpha}/k_\perp+iv_\perp J_n'(\xi)E_y(k_y)+v_zJ_n(\xi)E_z\right\}e^{-in\alpha+i\xi\sin\alpha},\quad(7)$$

where $v_{T\alpha}=\sqrt{T_\alpha/m_\alpha}$, $\xi=k_xv_\perp/\omega_{c\alpha}$, $J_n(\xi)=\xi^n/2^n n!$ is the Bessel function with $\xi<<1$, $J_n'(\xi)=dJ_n/d\xi$, $v_\perp$ is the component of the ion velocity perpendicular to the magnetic field, $n_{0\alpha}$ is the density of α-species resonant ions, T_α is their temperature, $\omega_{c\alpha}$ is the cyclotron frequency $\alpha=arctg(v_y/v_x)$, $L_B=(d\ln B_0/dx)$. The field *E (x)* is presented in the form of the Fourier integral

$$\mathbf{E}(x,t)=\int_{-\infty}^{\infty}e^{ik_xx}\mathbf{E}(k_x)dk_xe^{-i\omega t}\ .\quad(8)$$

We will assume the resonance $\omega=n\omega_{c\alpha}$ to occur at *x=0*, so that $\omega-n\omega_{c\alpha}(x)=-\omega(x/L_B)$, everywhere in (7), except the resonance denominator, $\omega_{c\alpha}$ is to be taken at the point *x=0*; in the resonance denominator in (7) we assume $Im\omega=+0$. With the account of (8) let us present the tensor $\hat{\varepsilon}_{ij}(x)$ in the form

$$\delta\hat{\varepsilon}_{ij}(x)E_j(x)=\int_{-\infty}^{\infty}e^{ik_xx}\delta\hat{\varepsilon}_{ij}(x,k_x)E_j(k_x)dk_x\ ,\quad(9)$$

where

$$\delta\varepsilon_{11}(x,k_x)=\frac{\omega_{p\alpha}^2L_B\omega_{c\alpha}}{2\pi\omega v_{T\alpha}^4 2^n n!k_x}\int_{-\infty}^{\infty}\frac{dv_y}{v_y-v_{res}(x)}e^{-v_y^2/2v_{T\alpha}^2+ik_xv_y/\omega_{c\alpha}}\int_{-\infty}^{\infty}dv_xv_xe^{-in\alpha-v_x^2/2v_{T\alpha}^2}\xi^n\ ,$$
$$\delta\varepsilon_{22}(x,k_x)=\frac{i\omega_{p\alpha}^4L_B}{2\pi\omega v_{T\alpha}^4 2^n n!}\int_{-\infty}^{\infty}\frac{dv_yv_y}{v_y-v_{res}(x)}e^{-v_y^2/2v_{T\alpha}^2+ik_xv_y/\omega_{c\alpha}}\int_{-\infty}^{\infty}dv_xe^{-in\alpha-v_x^2/2v_{T\alpha}^2}\xi^{n-1}v_\perp\ ,\quad(10)$$

$v_{res}(x)=\dfrac{\omega-n\omega_{c\alpha}(x)}{n}L_B$. Take into account that $\xi^n\exp(-in\alpha)=(k_x/\omega_{c\alpha})^n(v_x-iv_y)^n$, and perform in (10) the integration over $v_x'=v_x-iv_y$. This yields for $\delta\varepsilon_{11}(x,k_x)$, and $\delta\varepsilon_{22}(x,k_x)$ the expressions

$$\delta\varepsilon_{11}(x,k_x)=-i^n\frac{\omega_{p\alpha}^2|L_B|\sqrt{\pi}\xi^{n-1}}{\omega\omega_{c\alpha}\rho_\alpha n!2^{3n/2}}W(\zeta,\xi_1),$$
$$\delta\varepsilon_{22}(x,k_x)=-i^n\frac{\omega_{p\alpha}^2|L_B|\sqrt{\pi}\xi^{n-1}}{\omega\omega_{c\alpha}\rho_\alpha n!2^{3n/2}}W_1(\zeta,\xi_1),\quad(11)$$

where

$$W(\zeta,\xi_1)=\frac{1}{i\pi}\int_Ce^{-t^2+i\xi_1t}\frac{dt}{t\mp\zeta}\left[tH_n(-t)+\frac{1}{2}H_{n+1}(-t)\right],$$
$$W_1(\zeta,\xi_1)=\frac{1}{i\pi}\int_Ce^{-t^2+i\xi_1t}\frac{dt}{t\mp\zeta}tH_n(-t),\quad(12)$$
$$\zeta=-x/\sqrt{2}\rho_\alpha\ ,\ \xi=\xi_1/\sqrt{2}=k_x\rho_\alpha\ ,\ \rho_\alpha=v_{T\alpha}/\omega_{c\alpha}\ .$$

Here $H_n(t)$ are Hermite polynomials. Integration is performed along the contour C from $-\infty$ to $+\infty$ going around the singular point $\zeta=\pm\zeta$ from below, the upper sign relates to the case

$L_B>0$, whereas the lower sign is for $L_B<0$.

2. Solution of basic equations. If one neglects the effect of the resonance $\omega = n\omega_{c\alpha}$ on the FMSW propagation ($\delta\varepsilon_{ij}$ is small), then one obtains from equations (1) and (2) that

$$E_x(x) = -\frac{i\varepsilon_2}{\varepsilon_1} E_y(x), \tag{13}$$

where
$$E_y(x) = e^{\pm ik_0 x},\; k_0^2 = \frac{\omega^2}{c^2}\frac{\varepsilon_1^2 - \varepsilon_2^2}{\varepsilon_1}. \tag{14}$$

Accounting for small $\delta\varepsilon_{ij}$ and solving (2) and (1) by perturbation technique, we put at $x < x_0$, that

$$\boldsymbol{E} = \boldsymbol{E}^{(0)} + \boldsymbol{E}^{(1)} \quad , \; E_y^{(0)} = e^{ik_0 x} + re^{-ik_0 x}, \tag{15}$$

where $E^{(1)} << E^{(0)}$, r is the reflection coefficient, and at $x > x_0$, $E_y = Te^{ik_0 x}$, where T is the transmission coefficient of the wave through the layer. Choose the x_0 value considerably below the FMSW wavelength, $k_0 x_0 << 1$. Then for the perturbation $E_y^{(1)}$ we obtain the equation

$$\frac{d^2 E_y^{(1)}}{dx^2} = -k_0^2 E_y^{(1)} - \delta\hat{k}_+^2(x)e^{ik_0 x} - r\delta\hat{k}_-^2(x)e^{-ik_0 x}, \tag{16}$$

where
$$\delta\hat{k}_\pm^2(x)e^{\pm ik_x x} = \frac{\omega^2}{c^2}\left[\delta\varepsilon_{22}(x,k_x) - \frac{\varepsilon_2}{\varepsilon_1}\delta\varepsilon_{11}(x,k_x)\right]e^{ik_x x}\Bigg|_{k_x = \pm k_0} \tag{17}$$

Here the quantities $\delta\varepsilon_{ij}(x,k)$ are determined by formulas (11) and (12). Neglecting at $|\zeta| >> 1$ the quantities $\delta\varepsilon_{ij}$ we find that at $x < x_o$ $\boldsymbol{E} = \boldsymbol{E}^{(0)} + \boldsymbol{E}^{(1)}$, where $E_y^{(1)}$ is the induced solution of equation (16), equaling to

$$E_y^{(1)} = -1/\Delta \int_{-x_0}^{x} \delta k_+^2(x_1)dx_1 e^{ik_0 x} + 1/\Delta \int_{-x_0}^{x} \delta k_+^2(x_1)e^{2ik_0 x_1}dx_1 e^{-ik_0 x} -$$
$$- r/\Delta \int_{-x_0}^{x} \delta k_-^2(x_1)e^{-2ik_0 x_1}dx_1 e^{ik_0 x} + r/\Delta \int_{-x_0}^{x} \delta k_-^2(x_1)dx_1 e^{-ik_0 x} \quad , \tag{18}$$

where $\Delta = 2ik_0$. Equating the functions $E_y^{(0)} + E_y^{(1)}$ and $E_y = Te^{ik_0 x}$ and their derivatives $x = x_0$, we obtain two equations for the coefficients r and T. Hence we have

$$r = -\int_{-x_0}^{x_0} \delta k_+^2(x)e^{2ik_0 x}dx \left(\Delta + \int_{-x_0}^{x_0} \delta k_-^2(x)dx\right)^{-1}, \tag{19}$$

$$T = 1 - 1/\Delta \int_{-x_0}^{x_0} \delta k_+^2(x)dx + 1/\Delta^2 \int_{-x_0}^{x} \delta k_+^2(x)e^{2ik_0 x}dx \int_{-x_0}^{x} \delta k_-^2(x)e^{-2ik_0 x}dx. \tag{20}$$

For the absorption coefficient we obtain the expression

$$Q = 1 - |T|^2 - |r|^2 = 2\,\mathrm{Re}\,1/\Delta \int_{-x_0}^{x_0} \delta k_+^2(x)dx \quad . \tag{21}$$

On calculating the coefficient Q (21) we now take into account in expressions (11) and (12) the residue part of the integrals $W(\zeta,\xi_1)$ and $W_1(\zeta,\xi_1)$. Then for calculating the integral (21) over x we can take the following limits: $-\infty$ and $+\infty$. Developing $\exp(i\xi_1\zeta)$ in powers of $i\xi_1\zeta$ and taking into account that at $l+1<n$ the integrals containing ζ^{l+1} vanish, we keep in

this series the term with $l+1=n$. This results in the following

$$Q = Q_0 = \frac{\pi\xi^{2n-2}}{2^n n!} k_0 L_B \left(1 - \frac{\varepsilon_2}{\varepsilon_1}\right)^2 \frac{n^2 \omega_{p\alpha}^2}{k_0^2 c^2} . \qquad (22)$$

For the plasma consisting of one ion species this expression coincides at $n=2$ with one obtained in Ref. [1], and for arbitrary $n\geq 2$ it coincides with one from Ref.[2] In this case we have $\varepsilon_2/\varepsilon_1 = n$.

The presence of a nonuniformity leads to the reflection of the wave. From (19) we find for $\mathrm{Re}\, r$

$$|\mathrm{Re}\, r| = (1/2)Q . \qquad (23)$$

Conclusion. The presence of a narrow layer of the cyclotron resonance leads not only to the FMSW absorption determined by formula (21) but also to its reflection. As the layer is narrow, the reflection is weak; the energy reflection coefficient $|r|^2 \sim Q^2$ is small compared with Q. Formulas (23) and (24) for Q and r are obtained for small perturbations $\delta\varepsilon_{ij}$,

$$\eta(x) = \frac{|\delta\varepsilon_{ij}|}{|\varepsilon_{1,2}|} \sim \frac{(k_0\rho_\alpha)^{n-1} L_B}{2^{3n/2} n! \rho_\alpha} \frac{n_{0\alpha}}{n_0} << 1 . \qquad (24)$$

For the resonance n=1 and n=2 this condition holds only for minority ions with a very small concentration. For n≥3 this condition may hold even for $n_{0\alpha} \sim n_0$ because of the large numerical factor $n!$ in (24).

If $\eta >> 1$, then in the region where the damping is absent, $|\zeta| >> 1$, at $\varepsilon_1 + \delta\varepsilon_{11} \approx 0$ there occurs the conversion of FMSW into ion cyclotron oscillations or electrostatic ion-ion hybrid ones. The numerical calculations of Ref. [3] and the calculations of Ref.[4] by the narrow-layer technique are inapplicable because this conversion of FMSW into ion cyclotron oscillations in the fundamental resonance region for minority ions with the concentration exceeding that for bulk ions were not taken into account.

For the dispersion equation of Refs. [1,2] to be applicable, there must hold the condition of slow variation of the local wavenumber $1/k^2\,|k'| \sim 1/k_0\,|\eta'(x)| \sim |\eta|/k_0\rho_i << 1$. As $k_0\rho_\alpha << 1$, this condition is more severe than one employed in this paper for the applicability of the perturbation theory, $\eta << 1$. Apart from this inequality there must hold the condition $|A''| << |k'A|$, where the amplitude is $A \sim C(1+\eta(x))/\sqrt{k_0}$. Accounting for the fact that the quantity $\eta(x)$ changes over the distance $x \sim \rho_\alpha$, this condition has the form $k_0\rho_\alpha >> 1$, i.e., the WKB technique is inapplicable from the start to the treatment of the problem given, because in stating it one assumed that $k_0\rho_\alpha << 1$.

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