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Feedback Control of Drift Waves Turbulence and Chaos

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I. INTRODUCTION

Drift waves occur in magnetized plasmas i.e., in tokamaks and the ionosphere, and they provide one of the dominant mechanisms for particles and energy transport across magnetic fields lines. In a previous work [3] on the nonlinear evolution of the drift wave instability, the chaotic dynamics is analysed. This is done by a Galerkin approximation which reduces the Hasegawa–Wakatani model to a set of four ordinary differential equations (ODEs). Solving these equations, it's shown that near the threshold, the instability saturates in organized states throughout the interactions of a small number of modes. If we increase the number of unstable modes, the system becomes turbulent, and for a tokamak, leads to an anomalous transport with a diffusion coefficient larger than neoclassical one. Therefore reduction of turbulent transport is highly desirable and it can be achieved via control and suppression of fluctuations [4–9]. In this paper, as we look for a reduction of fluctuations, the expression *controlling chaos* defines the fact that the energy of chaotic or turbulent fluctuations is reduced. Here feedback control of drift waves fluctuations is simulated by introducing a term $-\alpha n$ in the density equation of the Hasegawa–Wakatani model, where n is the local density, α is proportional to the amplitude of the signal and the sign represents a phase shift.

II. DYNAMIC MODEL

The model we consider describes the nonlinear dynamics of dissipative drift wave turbulence. It consists of two partial differential equations describing the nonlinear evolution of the potential and density fluctuations, Φ and n, respectively. A mean density gradient dn_0/dx is assumed in the x direction. In dimensionless units, the equations are [1,2] :

$$\frac{\partial}{\partial t}(n-\rho^2\Delta_\perp\Phi)-\rho^2(\kappa\cdot\nabla)\Phi+\nu\Delta_\perp^2\Phi=\rho^2\{n-\rho^2\Delta_\perp\Phi,\Phi\} \tag{1}$$

$$\frac{\partial}{\partial t}\Delta_\perp\Phi+\omega_s\frac{\partial^2}{\partial z^2}(n-\Phi)=\rho^2\{\Delta_\perp\Phi,\Phi\} \tag{2}$$

where n and Φ are the density and potential fluctuations. Usual dimensionless variables have been used: $t\omega_{ci}\to t, r_\perp k_\perp\to r_\perp, zk_z\to z, \Phi=e\Phi/T_e, n=\delta n/n_0$. Here T_e is the electron temperature, ω_{ci} is the ion cyclotron frequency, δn is the deviation of density from the equilibrium n_0. Three parameters are involved in Eqs.(1),(2) : the hybrid *Larmor* radius $\rho=\rho_s k_\perp=k_\perp c_s/\omega_{ci}$, the transverse ion viscosity ν and $\omega_s=\omega_{ci}\omega_{ce}k_z^2/\nu_{ei}k_\perp^2$; $c_s^2=e/m_i$ is the ion sound speed, the labels e,i are related to electrons and ions, $\nu_{e,i}$ is the effective electron-ion collision frequency. In order to study chaos which occurs near the threshold, we use a Galerkin projection method. It consists in considering here a low level supercritical value of $R>1$ and restricts the problem to the dynamics of a few modes. It was shown that in the minimum harmonic approximation and in the frame of reference moving with velocity ω_d/k_y, the solution of Eqs.(1,2) slightly above the threshold $0<R-1\ll 1$ can be written under the form [3]

$$\Phi=\Phi_1\cos z+\Phi_2\cos 2z \tag{3}$$

with

$$\Phi_{1,2}=\sum_{m,n}A_{m,n}^{(1,2)}(t)\sin(mk_x x)\cos(nk_y y), \tag{4}$$

where only eight harmonics are present

$$A^{(i)}_{m,n} \equiv \{A^{(1)}_{10}, A^{(1)}_{11}, A^{(1)}_{20}, A^{(1)}_{21}, A^{(2)}_{10}, A^{(2)}_{11}, A^{(2)}_{20}, A^{(2)}_{21}\}. \tag{5}$$

Numerical simulation of the corresponding system of equations showed that amplitudes $A^{(1)}_{10}, A^{(1)}_{20}, A^{(2)}_{21}, A^{(2)}_{11}$ decay rapidly with time and only four amplitudes from the set (5) survive and define the supercritical regime. The corresponding equations are : [3]

$$\dot{X} = \nu(R-1)X + \frac{1}{2}fZU - XY \tag{6}$$

$$\dot{Y} = -eY + \frac{1}{2}X^2 \tag{7}$$

$$\dot{Z} = -\frac{e}{16}Z - \frac{1}{4}(1+f)XU \tag{8}$$

$$\dot{U} = f\nu(R-f)U + \frac{1}{2}XZ \tag{9}$$

with notations

$$R = a/\nu, \qquad f = 1 + 3k_x^2/k_y^2, \qquad d = k_x k_y \rho^2 \omega_d/\omega_s, \qquad e = \nu k_x^2/k_y^2, \tag{10}$$

$$X = A^{(1)}_{11}/d, \qquad Y = A^{(2)}_{20}/d, \qquad Z = A^{(2)}_{10}/d, \qquad U = A^{(1)}_{21}/d. \tag{11}$$

III. FEEDBACK CONTROL OF CHAOS

We use the simple numerical feedback method which acts on density, by the term $-\alpha n(t-\tau)$: the sign $-$ corresponds to a π phase shift in order to going down the level of fluctuations, α is proportional to feedback signal amplitude, τ is the time delay and n is the density fluctuations. This term is injected in the right part of Eq.(1) : and the system is modified so that we find with Galerkin approximation the following equations :

$$\dot{X} = \nu(R-1)X + \frac{1}{2}fZU - XY - jX(t-\tau) \tag{12}$$

$$\dot{Y} = -eY + \frac{1}{2}X^2 - 2jY(t-\tau) \tag{13}$$

$$\dot{Z} = -\frac{e}{16} - \frac{1}{4}(1+f)XU - 2jZ(t-\tau) \tag{14}$$

$$\dot{U} = f\nu(R-f)U + \frac{1}{2}XZ - fjU(t-\tau) \tag{15}$$

where $j = \alpha\omega_s/4\omega_d\rho^2 k_x k_y$ is proportional to α. Let us study the effects of feedback on chaos, first with time delay τ=0. With no time delay, the control is ideal. The idea is to adjust the value of j so that it exactly cancels the growth rate and reduces the chaotic fluctuations. If we consider the system (12-15), the positives growth rates are for X and U modes so we will focus on these ones.

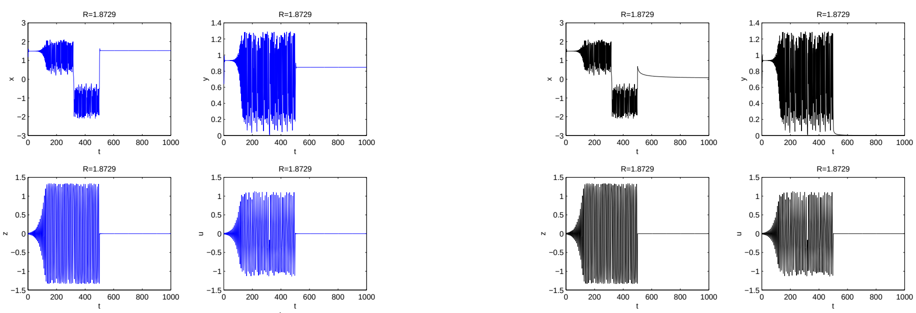

FIG. 1. Amplitudes in chaotic case a)without time delay, we begun feedback control at time t=500, j=(R-f)ν (U growth rate set to zero) and b)j=(R-1)ν (X growth rate set to zero).

This figure shows the effect of numerical feedback control which starts at time t=500 in killing X growth rate. We have suppressed drift wave amplitude for the four modes (X,Y,Z,U), and the result is the feedback control succeed. What is important is the fact that here τ=0, we are in an ideal case, in order to be the closest to a physical situation we have to introduce a non–zero time delay.
Now the same work is done with a non–zero time delay :

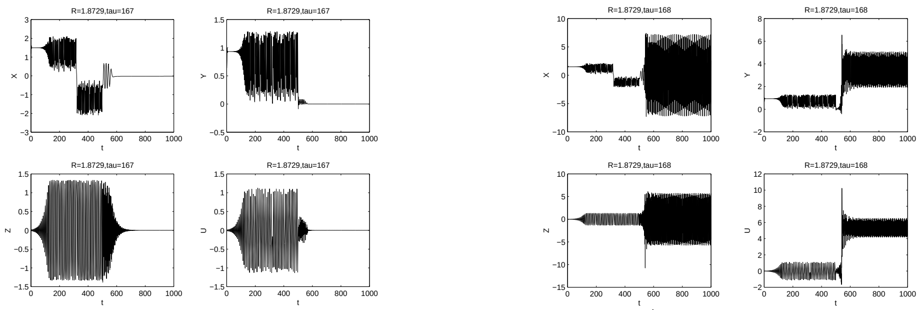

FIG. 2. Amplitudes in chaotic case with a time delay of a) $t = 0.835\omega_{drift}^{-1}$, feedback begun at time t=500 and control is successful, b)with a time delay of time delay of $t = 0.84\omega_{drift}^{-1}$, here fluctuations exploded.

Fig.3 shows that until a value of $\tau = 0.835\omega_{drift}^{-1} \sim \omega_{drift}^{-1}$ the control succeeds, there is a damping at the beginning of the feedback at time t=500, so this particular value is what we call : maximum time delay. In the second plot of the Fig.3 the value of time delay is $\tau = 0.84\omega_{drift}^{-1}$, the fluctuations level blows up to saturate but with a level of amplitude higher than initial signal. Why this difference ? In order to understand it, we must calculate auto-correlation time of the initial signal (i.e.,without the $-\alpha n$ term). The auto-correlation function is defined by

$$c(T) = \frac{\int f(t)f(t+T)dt}{\int \mid f \mid^2 dt} \tag{16}$$

where f takes successively the values X,Y,Z,U in the chaotic case. The results give an auto-correlation time (τ_{corr}) of the order of ω_{drift}^{-1}, close to time delay. If, like in Fig.9 the time delay is larger than auto-correlation time, the non−perturbed signal and feedback signal are not correlated so that control is not efficient. While in Fig.4 maximal time delay is larger than correlation one, the two are correlated and the control succeed. We have to notice that the range of time delay τ_{delay} is from 0 to 5, i.e., from zero to 0.835 control succeed, and after 0.840 to 5 or more amplitudes explode. The main result for feedback control of chaos is that $\tau \leq \omega_{drift}^{-1} \sim \tau_{corr}$. What is important to notice here is that such a small number of unstable modes (four) and the fact that control succeed or fail if we change the time delay from $\tau_{delay} = 0.835\omega_{drift}$ to $\tau_{delay} = 0.840\omega_{drift}$ is not physically reachable in an experiment. Our model is theoretical and relevant for the evolution to the turbulence.

IV. FEEDBACK CONTROL OF TURBULENCE

In order to describe a drift wave turbulence, we simulate the system Eqs.(1,2) known as Hasegawa-Wakatani model. The difference with previous case is that the number of degrees freedom increase (with a control parameter R$\simeq$35, so that the Galerkin approximation becomes questionable. Then we simulate feedback control of turbulence introducing the $-\alpha n$ term in the full system (density equation (Eq.(1)) like in the chaotic case but here without a Galerkin reduction. :
First of all let us study the case without time delay :

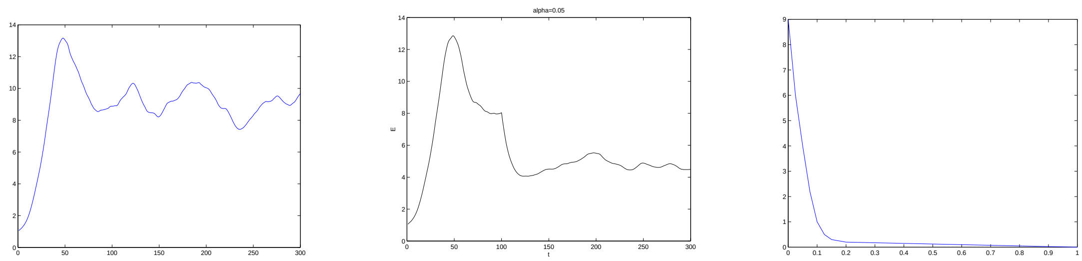

FIG. 3. Time evolution of fluctuation energy without time delay a)without feedback, fluctuation energy in turbulent case for a value of $\alpha = 0.05$, b)feedback begun at time t=100, c)energy fluctuations function of α parameter.

when feedback is activated, i.e., at time t=100, we indicate the final level of saturation for each α. The two first plots in fig.4 show a typical time evolution : in the first fifty time steps the amplitude increases exponentially. It corresponds to the linear part. As the regime of linear saturation is needed, the feedback is set on, leading to a saturation around an average value. In a second stage, we report these values in the last plot of the Fig.4 where is presented the level of saturation as a function of the α parameter. This plot shows how feedback amplitude make decrease the energy fluctuations, we distinguish two ranges : the first from α=0.2 to 1 where α weakly influences the level of fluctuations and the second one from α=0 to α=0.2 the action of feedback is very strong; in this small window of α, the level of fluctuations is reduced by a factor twenty ! The result is that we only need a small feedback amplitude (typically 10 percent) to strongly reduce turbulence. Now we are looking at feedback control of turbulence with time delay, which is more physical : in this paragraph we had to find the maximal time delay τ of this system, i.e., if $t < \tau$ the level of fluctuations is below the initial saturation without time delay and conversely.

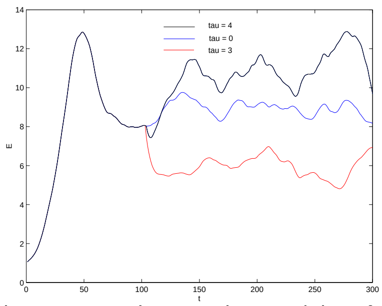

FIG. 4. Fluctuation energy in turbulent case with several time delay, feedback begun at t=100, time units are in drift frequency.

In the fig.5, the signal of feedback begins at t=100 and fixing a value of α, we have several values of τ : For $\tau = 0$ feedback is ideal, until $\tau = 3$ the level of saturation is below ideal feedback, the control succeeds. For $\tau = 4$ or more, the level is higher than ideal feedback, the control failed. Let us compare this value with the correlation time as in Sec. III. We find that the correlation time is : $\tau_{corr} \simeq \tau_{delay} \simeq \omega_{drift}^{-1}$. This result shows the efficiency of this method on controlling dissipative drift wave turbulence. We have seen for feedback control of chaos that fluctuations are reduced to zero if the maximum time delay is not larger than the correlation time, i.e., ω_{drift}^{-1}. For feedback control of turbulence, the fluctuations are highly reduced and the maximum time delay is of the order of the drift frequency. However, the correlation time is : $\omega_{corr} > \omega_{delay} \simeq \omega_{drift}^{-1}$. In the two situations the main result is that, as correlation time is larger or equal to maximal time delay, the control via a feedback method is theoretically efficient. Once again these numerical results have been obtained with assumptions like a feedback done on the total density, the non introduction of noise, so that these results are a first step to a situation closer to experiment, and these hypothesis will be taken in account in a future work.

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