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Electron Cyclotron Current Drive with a Symmetric Spectrum in a Tokamak

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It is shown that RF waves need not have an asymmetric spectrum in order to drive current in a toroidal plasma if the wave field is instead up-down asymmetric and the plasma is not entirely collisionless. For instance, pure electron cyclotron heating drives a toroidal current even if there is no net wave-particle momentum transfer and the wave field does not interact preferentially with electrons travelling in one direction. The resulting current drive efficiency is calculated and is found to be smaller than that of the conventional current drive mechanism in the banana regime, but not insignificant in the plateau regime.

Current drive in plasmas by radio-frequency (RF) waves usually requires these waves to have an asymmetric spectrum, so that they either interact preferentially with electrons travelling in one direction along the magnetic field or impart net parallel momentum to the electrons [1]. This creates an asymmetry in the electron distribution function and thus produces an electric current parallel to the field. Here, we demonstrate, somewhat surprisingly, that in a toroidal plasma no such spectral asymmetry is necessary for current drive in a plasma where collisions occur, if the wave field is instead up-down asymmetric. In practice this tends to be the case, in particular for electron cyclotron current drive (ECCD) in tokamak experiments. For instance, in the Doublet III-D tokamak, the waves are believed to be absorbed entirely above the magnetic midplane [2]. Our calculations are focused on this case of ECCD in tokamaks, but the basic physical mechanism is more general.

In order to understand how the current is produced, consider a toroidal plasma with localized electron cyclotron resonance heating (ECRH) somewhere above the magnetic midplane. As an electron is heated when passing through the resonance, its magnetic moment $\mu = mv_\perp^2/2B$ increases. This increases the mirror force $F_\| = -\mu\nabla_\| B$, and if, say, $\nabla_\| B$ is positive at the resonance this reduces the parallel velocity of the electron and thus produces a positive electric current. Once the electron has travelled half a poloidal turn around the flux surface, the mirror force has changed sign and the effect of the increased magnetic moment is reversed. However, by this time collisions will partially have restored μ to its former value, and a net effect thus persists. It follows from this physical picture that the present current drive mechanism requires both toroidicity and collisions. The current drive efficiency will therefore be poor when either the collisionality $\nu_* = \nu_e qR/v_{Te}\varepsilon^{3/2}$ or the inverse aspect ratio $\varepsilon = r/R$ are very small. (Here q is the tokamak safety factor, ν_e the electron collision frequency, and v_{Te} the electron thermal speed.) Thus, the efficiency is low in the centre of a typical hot tokamak plasma, but can be significant further away from the magnetic axis where both the collision frequency and ε are relatively large. The present mechanism might therefore be helpful for current-profile control in advanced tokamaks, which is frequently required some distance away from the centre.

We begin our analysis by considering the banana regime, $\nu_* \ll 1$. For simplicity, we assume that the wave field is sufficiently weak that the electron population remains close to local thermodynamic equilibrium. The electrons are then described by the linearized drift kinetic equation

$$v_{||}\nabla_{||} f_1 - C(f_1) = -\mathbf{v}_d \cdot \nabla f_0 + Q + S, \tag{1}$$

where $\mathbf{v}_d$ is the drift velocity and C is the Coulomb collision operator linearized around a Maxwellian, f_0. The wave-particle interaction is described by the quasilinear operator

$$Q(\theta) = \frac{1}{v_\perp}\frac{\partial}{\partial v_\perp} v_\perp \hat{D}(\theta,\mathbf{v})\frac{\partial f_0}{\partial v_\perp}, \tag{2}$$

where $\hat{D} = D\delta(\theta - \pi/2)x_\perp^{2(l-1)}$ for ECRH at the l:th harmonic right above the midplane, D is a constant, θ the poloidal angle, and we have introduced the dimensionless velocity vector $\mathbf{x} = \mathbf{v}/v_{Te}$, with $v_{Te} = (2T_e/m_e)^{1/2}$ the thermal speed. Note that this operator describes heating in the perpendicular direction, but operates symmetrically with respect to the parallel direction. Thus, the wave field does not impart any net momentum to the plasma and does not interact preferentially with electrons travelling in any particular parallel direction. On the right-hand side of (1) there also appears a term S, which accounts for any additional sources and losses, e.g., caused by anomalous transport across the confining magnetic field.

For simplicity, we assume that the aspect ratio is large, $\varepsilon \ll 1$, the flux surfaces circular, and the effective ion charge high, $Z_{\text{eff}} \gg 1$, so that electron-electron collisions can be ignored and the collision operator becomes

$$C(f_1) = \frac{\nu_{ei}(v)}{2}\frac{\partial}{\partial\xi}\left(1-\xi^2\right)\frac{\partial f_1}{\partial\xi}, \tag{3}$$

where $\nu_{ei} = 3\pi^{1/2}/4\tau_{ei}x^3$ is the electron-ion collision frequency and $\tau_{ei} = 3(2\pi)^{3/2}\varepsilon_0^2 m_e^{1/2} T_e^{3/2} /n_e Z_{\text{eff}}^2 e^4 \ln\Lambda$ the collision time. It is useful to split the heating operator into terms that are even and odd in the poloidal angle $-\pi < \theta \leq \pi$, $Q(\theta) = Q_+(\theta) + Q_-(\theta)$, where $Q_\pm(\theta) = [Q(\theta) \pm Q(-\theta)]/2$. Since (1) is a linear equation and the driving terms on the right appear additively, the solution f_1 consists of a sum of the separate contributions from these terms. Our primary interest here is to find the current driven by the up-down asymmetric drive $Q_-(\theta)$. This current is normally neglected in theories of ECCD proceeding from the orbit average of the kinetic equation (1) since this average of $Q_-(\theta)$ vanishes. Thus, we only include the asymmetric piece

$$Q_-(\theta) = \frac{m_e D}{T_e}\left[\delta\left(\theta - \frac{\pi}{2}\right) - \delta\left(\theta + \frac{\pi}{2}\right)\right] x_\perp^{2(l-1)}\left(x_\perp^2 - l\right) f_0$$

of the heating operator (2) on the right-hand side of (1). This term can be written as $Q_-(\theta) = v_{||}\nabla_{||}h$, where the gradient is taken at fixed x and Λ, and h is defined by

$$h = -\frac{2qR_0 D f_0}{v_{Te}^3}\left(x^2\Lambda - l\right)\frac{x^{2(l-1)}\Lambda^{l-1}}{\sigma\sqrt{1-\Lambda}} H_{\text{out}}(\theta), \tag{4}$$

for $\Lambda < 1$ and $h = 0$ for $\Lambda > 1$. Here $\Lambda = v_\perp^2 B_0/v^2 B$, $\sigma = v_{||}/|v_{||}|$, and B_0 is a reference magnetic field, which we choose to be the field at the poloidal location of the heating, $\theta = 90°$. R_0 is defined by $qR_0 = 1/\nabla_{||}\theta$ at $\theta = \pi/2$ and becomes equal to the major radius of the magnetic axis in a standard, large-aspect-ratio, circular equilibrium. The even function $H_{\text{out}}(\theta)$ is defined by

$H_{\rm out}(\theta) = H(\theta + \pi/2) - H(\theta - \pi/2)$, where H is the Heaviside function. The kinetic equation now assumes the form

$$v_{\|}\nabla_{\|}(f_1 - h) = C(f_1),$$

which is familiar from neoclassical transport theory [3]. Expanding in the smallness of the collision frequency in the usual way, $f_1 = f_1^0 + f_1^1 + \ldots$, gives, in lowest order, $f_1^0 = g + h$, where g only depends on constants of motion, $g = g(v, \Lambda, \psi, \sigma)$. It follows from a conventional argument that g vanishes in the trapped domain, and is determined by the constraint $\left\langle (B/v_{\|})C(g+h) \right\rangle = 0$ in the passing domain. If the Lorentz approximation (3) is used for the collision operator, this equation is easily integrated once to give

$$\frac{\partial g}{\partial \Lambda} = -\frac{2qR_0 D f_0}{v_{Te}^3} \frac{\langle v_{\|} H_{\rm out}(\theta)\rangle}{\langle v_{\|}\rangle} \frac{x^{2l-3}\Lambda^{l-1}}{\sigma\sqrt{1-\Lambda}} \left[x^2 + \left(x^2\Lambda - l\right)\left(\frac{l-1}{\Lambda} + \frac{1}{2(1-\Lambda)}\right)\right]. \qquad (5)$$

The current carried by h vanishes and that carried by g is given by

$$j_{\|} = 2\pi e B \int_0^\infty v^3 dv \int_0^{1-\varepsilon} \Lambda \frac{\partial g}{\partial \Lambda} d\Lambda, = 0.99\,\Gamma\left(l + \frac{1}{2}\right) \frac{q e n_e m_e R_0 D}{2\pi \varepsilon^{1/2} T_e}, \qquad (6)$$

where n_e is the electron density. The flux surface average of the heating power density is

$$\langle P \rangle = \left\langle \int \frac{m_e v^2}{2} Q_+(\theta) d^3v \right\rangle = \frac{\Gamma(l+1) m_e n_e |D|}{\pi}, \qquad (7)$$

so the local current drive efficiency becomes

$$\frac{|j_{\|}|}{\langle P \rangle} = 0.99\,\frac{\Gamma(l+1/2)}{2\Gamma(l+1)} \frac{e q R_0}{\varepsilon^{1/2} T_e}. \qquad (8)$$

A widely used figure of merit for current drive efficiency is $\eta = n_e I R_0 / P_{\rm ECRH}$, where I is the total current driven and $P_{\rm ECRH}$ the total power deposited by the waves, so that $I/P_{\rm ECRH} = \left\langle j_{\|} \right\rangle / 2\pi R_0 \langle P \rangle$ if the current drive occurs only in the vicinity of one particular flux surface [4]. For conventional current drive, η is of the order

$$\eta_0 = \frac{n_e e \tau_{ei}}{2\pi\sqrt{2 m_e T_e}} = \frac{18}{\ln\Lambda} \frac{T_{\rm keV}}{Z_{\rm eff}^2}\, 9 \cdot 10^{17}\ {\rm A/Wm^2},$$

where $T_{\rm keV}$ is the electron temperature in keV. In contrast, the result (8) can be expressed as

$$\eta_{\rm ban} = 0.99 \frac{\Gamma(l+1/2)}{\Gamma(l+1)}\, \varepsilon \nu_* \eta_0, \qquad (9)$$

where both the collisionality $\nu_* = qR_0/(v_{Te}\tau_{ei}\varepsilon^{3/2})$ and ε have been assumed to be small. The current drive efficiency is therefore modest in the banana regime.

We now turn our attention to the plateau regime, which is defined by $1 \ll \nu_* \ll \varepsilon^{-3/2}$. To calculate the current drive efficiency in this regime, we find it more convenient to use the adjoint method [5] than to solve (1) directly. In this method, the adjoint equation,

$$v_{\|}\nabla_{\|} G + C(G) = v_{\|} f_0, \qquad (10)$$

is first solved, and the current is then calculated from

$$\left\langle j_{\|}\right\rangle = e\left\langle \int \frac{G}{f_0} Q_{-}(\theta)\, d^3v \right\rangle. \tag{11}$$

To solve the adjoint equation (10), it is convenient to write $G = -v_{\|} f_s(v) + k$, where f_s is the Spitzer function defined by $C(v_{\|} f_s) = -v_{\|} f_0$. The equation for k then becomes

$$v_{\|}\nabla_{\|}k + C(k) = v_{\|}\nabla_{\|}(v_{\|} f_s) = -\frac{\varepsilon v_{\perp}^2}{2qR_0}\, f_s(v)\, \sin\theta,$$

which has a well-known solution in the plateau regime [3]. Only the even (in $v_{\|}$) part of k contributes to the current (11) and is given by $k_{\text{even}} = (\pi/2)\varepsilon v\, f_s(v)\, \sin\theta\, \delta(v_{\|}/v)$ in the plateau limit $2v/\nu_{ei}qR_0 \to \infty$. Inserting this result in (11) shows that the current associated with k arises from the up-down asymmetry of the heating and becomes

$$j_{\|} = \frac{5}{3\pi}\,\Gamma\left(l + \frac{5}{2}\right)\sqrt{\frac{2m_e}{T_e}}\,\varepsilon n_e e \tau_{ei} D$$

in the Lorentz limit $Z_{\text{eff}} \gg 1$, where the Spitzer function is equal to $f_s(v) = f_0(v)/\nu_{ei}(v)$. The current drive efficiency is obtained by dividing this result by the heating power (7),

$$\frac{|j_{\|}|}{\langle P\rangle} = \frac{5\sqrt{2}}{3}\,\frac{\Gamma(l+5/2)}{\Gamma(l+1)}\,\frac{\varepsilon e \tau_{ei}}{\sqrt{m_e T_e}}.$$

In terms of the parameter η the efficiency is

$$\eta_{\text{plat}} = \frac{10}{3}\frac{\Gamma(l+5/2)}{\Gamma(l+1)}\,\varepsilon\eta_0.$$

For instance, $\eta_{\text{plat}} \simeq 11\varepsilon\eta_0$ for heating at the first harmonic ($l = 1$), and $\eta_{\text{plat}} \simeq 19\varepsilon\eta_0$ at the second harmonic ($l = 2$). This suggests that the current drive efficiency can be substantial in the plateau regime although ε is formally a small parameter.

The RF-driven current $j_{\|}$ is in the same direction as the total plasma current if the electron drift is directed toward the heated side of the flux surface, and $j_{\|}$ opposes the total current if the drift is the opposite direction. Recently, experiments in the DIII-D tokamak [2] have indicated that the measured current drive efficiency exceeds the classical prediction. It appears that the effect we have considered may be too small to explain this discrepancy, but it should be noted that the error bars associated with the experiment are substantial. In addition, there is the theoretical uncertainty of how the plateau and banana regimes should be matched to each other. The direction of the wave-driven current calculated here appears to coincide with that in the DIII-D experiments. The current drive efficiency should therefore drop if the toroidal field is reversed.

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