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Alfvén eigenmode structure in Helias configurations

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Introduction. Alfvén Eigenmodes (AE) of toroidal plasmas are studied rather well but mainly for tokamaks; stellarators received much less attentions. Meanwhile, one can expect that specific instabilities associated with the gaps in Alfvén continuum which are absent in tokamaks may arise in stellarators, in particular, in the optimized Wendelstein-line stellarators (or Helias configurations). Indeed, a Helias is characterized by, first, strong elongation of the plasma cross section rotating along the large azimuth of the torus, second, the presence of several dominant Fourier harmonics of the magnetic field strength, and third, the small magnetic shear. Due to these facts, AEs in the Wendelstein stellarators may have a number of peculiarities making these modes different from those not only in tokamaks but also in stellarators of other types. This motivated the fulfillment of the present work aimed at studying possible Alfvén eigenfrequencies and eigenfunctions in Helias configurations, in particular, in a Helias reactor [1]; such a study is necessary for a subsequent investigation of Alfvén instabilities.

Basic equation. The optimized stellarators are characterized by the almost vanishing bootstrap current. Taking this into account and neglecting effects of plasma pressure, we obtained the following equation for shear AEs in Helias configurations:

$$\hat{L}\frac{\partial}{\partial x^j}\left(\nabla_\perp \hat{L}\Phi\right)^j + \omega^2 R_0^2 \frac{\partial}{\partial x^j}\frac{(\nabla_\perp\Phi)^j}{\bar{v}_A^2 h^4} = 0, \qquad (1)$$

where Φ is scalar potential of the electric field perturbations, $x^j = \psi, \theta, \phi$ are the Boozer coordinates, the subscripts and superscripts j label the covariant and contravariant vector components, $\hat{L} = \iota\partial/\partial\theta + \partial/\partial\phi$, ι is the rotational transform, $\nabla_\perp = \nabla - \vec{b}(\vec{b}\cdot\nabla)$, $\vec{b} = \vec{B}_0/B_0$, $\vec{B}_0$ is the equilibrium magnetic field strength, ω is the wave frequency, $\bar{v}_A = \bar{B}/(4\pi\rho)$, $\bar{B} = B_0/h_B$, $h_B = 1 + \Sigma_{\mu\nu}\epsilon_B^{(\mu\nu)}\cos(\mu\theta - \nu N\phi)$, μ and ν are integers, R_0 is the large radius of the torus. Equation (1) is three-dimensional. It can be presented as an infinite set of the second-order one-dimensional differential equations by expanding the perturbation $\widetilde{\Phi}$ in a Fourier series, $\Phi = \sum_{m,n}\Phi_{mn}(\psi)\exp(im\theta - in\phi - i\omega t)$ (m and n are the poloidal and toroidal mode numbers), as follows:

$$\sum_{\mu,\nu}\left[\frac{\partial}{\partial r}r^3\mathcal{K}^2\frac{\partial E_{m,n}}{\partial r} + QE_{m,n} + \frac{\partial}{\partial r}r^3\left(\mathcal{E}_+\frac{\partial E_{m+\mu,n+\nu N}}{\partial r} + \mathcal{E}_-\frac{\partial E_{m-\mu,n-\nu N}}{\partial r}\right)\right] = 0, \qquad (2)$$

where $E = \Phi/r$, r is the flux surface radius defined by $\psi = \bar{B}r^2/2$, $\mathcal{K}^2 = \omega^2/\bar{v}_A^2 - k_{m,n}^2$, $Q_{m,n} = r\mathcal{K}^2(1-m^2)+r^2\omega^2 d\bar{v}_A^{-2}/dr$, $\mathcal{E}_\pm = (\omega^2/\bar{v}_A^2)\left(\epsilon_g^{(\mu\nu)}/2 - 2\epsilon_B^{(\mu\nu)}\right) - k_{m,n}k_{m\pm\mu,n\pm\nu N}\epsilon_g^{(\mu\nu)}/2$, $k_{m,n} \equiv k_{||}(m,n) = (m\iota - n)/R_0$, $\epsilon_g^{(\mu\nu)}$ are the quantities determined by the metric tensor $g^{\psi\psi}$ according to $g^{\psi\psi} = 2\psi\bar{B}\delta_0\left[1 + \Sigma_{\mu\nu}\epsilon_g^{(\mu\nu)}\cos(\mu\theta - \nu N\phi)\right]$, δ_0 is average elongation of the plasma cross section, N is the number of the field periods.

The structure of Alfvén continuum. The variety of coupling parameters leads to a number of gaps in Alfvén continuum where discrete modes can reside. In particular, in a Helias the dominant coupling parameters are $\epsilon_g^{(21)}$ producing the gap with HAE_{21} modes (HAE means helicity-induced AE), $\epsilon_g^{(22)}$ resulting in HAE_{22} modes, $\epsilon_g^{(20)}$ resulting in EAEs, $\epsilon_g^{(01)}$ and $\epsilon_B^{(01)}$ producing MAEs (mirror-induced AEs), $\epsilon_B^{(11)}$ resulting in HAE_{11} modes, and $\epsilon_B^{(10)}$ producing TAEs. The gaps are located in the vicinity of the wave frequencies $\omega^{(\mu\nu)}$ and the radii $r_*(\mu,\nu)$ determined from the condition that two cylindrical continua intersect. These frequencies are given by

$$\omega^{(\mu\nu)} = (N\nu - \mu\iota_*)\frac{\bar{v}_A(r_*)}{2R_0}, \quad \iota_* \equiv \iota(r_*) = \frac{2n + \nu N}{2m + \mu} \tag{3}$$

and shown in Fig. 1. A simple estimate based on the assumption that the gaps are independent yields the width of a gap $\Delta\omega = \left|\epsilon_B^{(\mu\nu)} - \epsilon_g^{(\mu\nu)}/2\right|$, which may exceed the distance between the characteristic frequencies $\omega^{(\mu\nu)}$. This implies that the gaps cannot be treated independently. A corresponding analysis taking into account the gap interaction shows that the gaps "repel" each other, tending to prevent their overlap.

To obtain the structure of the continuum gaps more definitely, we have developed a method for finding the structure of the shear Alfvén continuum in a general three-dimensional toroidal magnetic configuration. It is based on the fact that ω belongs to continuous spectrum of Eq. (1) if there is a radial point ψ at which ω is an eigenvalue of the equation

$$\hat{L}\left(g^{\psi\psi}\hat{L}\Phi\right) + \omega^2 R_0^2 \frac{g^{\psi\psi}}{\bar{v}_A^2 h^4}\Phi = 0 \tag{4}$$

with the natural boundary condition of periodicity. This approach was implemented in the computer code COBRA (COntinuum BRAnches).

The obtained structure of continuum gaps for a Helias is presented in Fig. 2. We observe that, first, the actual width of the gaps is less than that obtained with neglecting the gap interaction, and, second, most gaps are shifted upward or downward from their expected position. The relative magnitude of the "compression" and the shift are the strongest for the gaps located near the HAE_{21} gap, which indicates the strong influence of the HAE_{21} gap on its neighbours. An interesting feature of the spectrum in Fig. 2 is the presence of gaps with coupling numbers (μ,ν) which are present neither in $g^{\psi\psi}$ nor in B, which indicates that the three- (or more) mode interaction plays an important role.

Discrete eigenmodes. To solve the eigenvalue problem, we have developed the BOA (Branches Of Alfvén modes) code. Using this code, we have investigated the high-frequency part of the Alfvén spectrum, namely, the HAE_{21} and MAE modes, in a Helias reactor[1]. We took the plasma mass density profile in the form $\rho(r) = \rho(0)\times\left[1 + (r/(ax_n))^{10}\right]^{-1}$, where x_n is a parameter, a is the plasma radius. Note that the considered case corresponds to the high-iota high-mirror version of W7-X studied in Ref. [2]

(where, however, only homogeneous plasma was considered) with using the CAS3D3 code.

In studying the HAE_{21} modes we neglected the effect of other kinds of AEs, which is justified due to the large magnitude of $\epsilon_g^{(21)}$. In addition, we restricted ourselves to analysis of the two-wave interaction [the (m, n) and $(m + 2, n + N)$ modes]. We found that the gap is closed for $x_n \leq 0.85$. This implies that the gap is open provided that $n(a)/n(0) \geq 1/6$. There are discrete modes inside the gap, shown, in particular in Fig. 3. The fact that the eigenfrequency intersects the continuum manifests itself only in small spikes near the plasma edge. Lowering x_n, we found that global smooth eigenfunctions disappear when $x_n < 0.9$.

The MAE gap is strongly affected by other gaps. Therefore, we considered the interaction of more than two waves. Namely, in addition to the modes (m, n) and $(m, n + N)$, we considered the modes $(m - 2, n)$ and $(m + 2, n + N)$, taking into account the presence of the coupling parameters $\epsilon_B^{(01)}$, $\epsilon_g^{(01)}$, $\epsilon_g^{(21)}$, $\epsilon_g^{(20)}$. Like in the case of HAE_{21}, we obtained a pair of eigenfunctions (with opposite and coinciding phases of the main components) for each set of mode numbers. Figure 4 gives an example of MAEs.

We considered also the Global AEs (GAE) in a Helias reactor, restricting ourselves to the cylindrical approximation. We found that discrete solutions may exist, the results being rather sensitive to the magnetic shear.

Summary. In this work, an equation of shear Alfvén eigenmodes (AE) for Helias configurations is derived. The metric tensor coefficients, which are contained in this equation, are calculated analytically. Two numerical codes are developed: the first one, COBRA (COntinuum BRAnches), is intended for the investigation of the structure of Alfvén continuum; the second, BOA (Branches Of Alfvén modes), solves the eigenvalue problem. The family of possible gaps in Alfvén continuum of a Helias configuration relevant to W7-X and a Helias reactor is obtained. It is shown that the multiple-wave interaction plays an important role, affecting both the width and the location of gaps and even producing new gaps. It is found that the largest gap is characterized by the coupling numbers $\mu = 2, \nu = 1$ and associated with rotation of the plasma cross-section along the large azimuth of the torus. The presence of discrete eigenmodes in the high-frequency part of the Alfvén spectrum, namely, HAE_{21} and MAE modes, is predicted. An important role of plasma inhomogeneity for the gap modes is revealed. In addition to the AEs residing in the continuum gaps, the GAE modes are considered.

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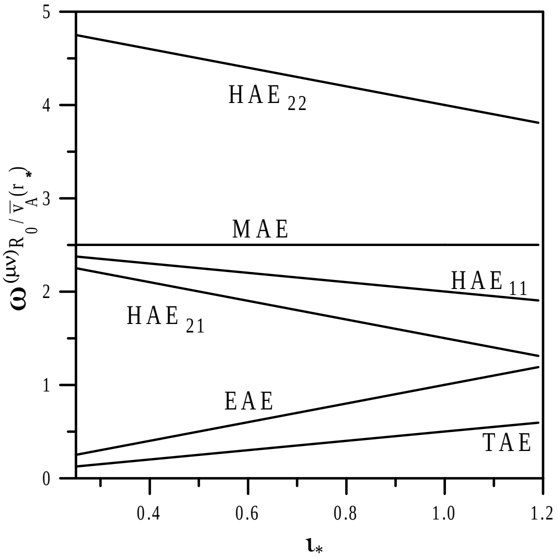

Fig.1.

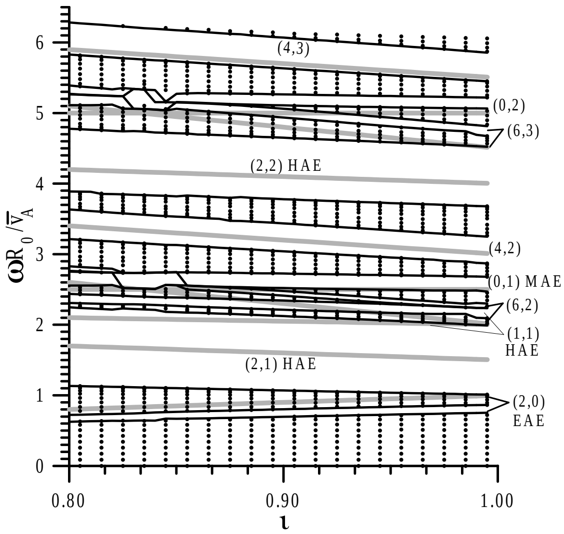

Fig.2.

Fig.1. Characteristic frequencies of various kinds of AEs according to Eq. (3). The region $0.8 \leq \iota_* \leq 1$ *corresponds to a Helias reactor.*
Fig.2. Gaps in the Alfvén continuum. The gaps are labeled by the coupling numbers (μ, ν) *responsible for the formation of each gap. Dots, regions of continuum; thin lines, the calculated "banks" of the gaps; wide grey lines, the places where the same gaps would be located if* $\epsilon \to 0$.

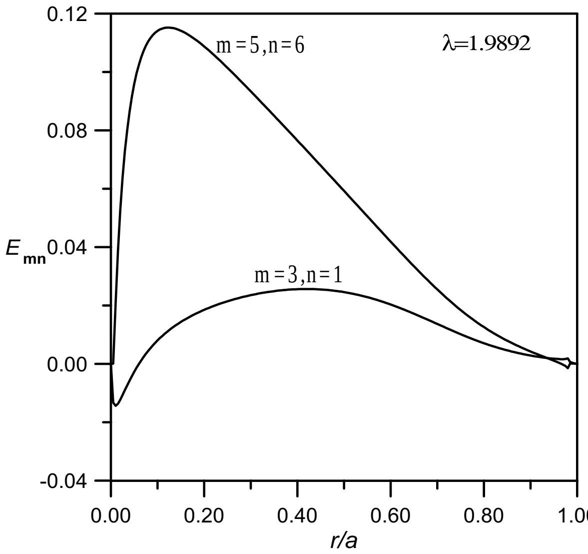

Fig.3.

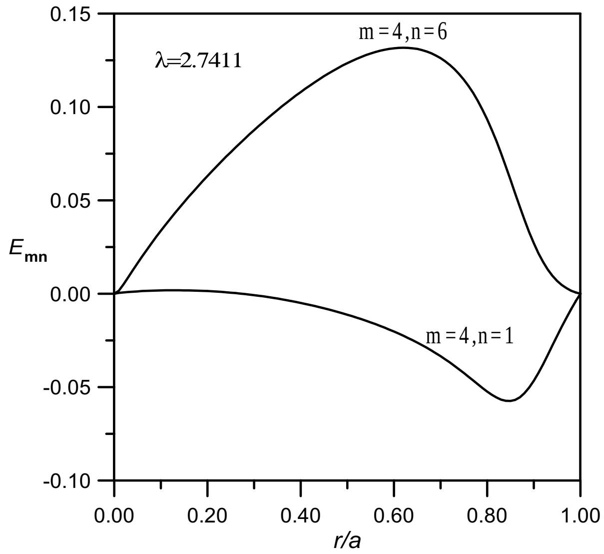

Fig.4.

Fig.3. The HAE_{21} *radial structure in a Helias reactor for* $x_n = 0.9$. $\lambda = \omega R_0/\bar{v}_A(r=0)$ *is the normalized eigenfrequency.*
Fig.4. The MAE radial structure in a Helias reactor for $x_n = \infty$.