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STOCHASTICTRANSPORTOFFASTIONSINSTELLARARTORS

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1. INTRODUCTION

Stochastic collisionlesstransportisknowntocontributesignificantlytothelossof energeticionsin toroidalmagneticconfinementdeviceseveninthecaseofa tokamakwitha weakaxialasymmetryofmagneticfield[1].Helicalripplesofa stellaratormagneticfield considerablycomplicatethefastparticlemotionascomparedtothatinthe tokamak,andmay causethestochastic behaviourand stochasticityinduced collisionlesslossmechanismsof highenergyparticlesinthehelicalconfinementsystemsaswell.Thusthemotionofenergetic toroidallytrappedparticleswiththe toroidalprecessionbeingintheresonancewiththe helicalrippleperturbationsbecomesstochasticduetothenon-conservationofthe longitudinaladiabaticinvariant $J=\int V_{\|}ds$ [2].

Themainpurposeofthepresentpaperisthestudyofthestochastictransportoffast ionsin stellaratorsinducedbythenon-conservationofthemagneticmoment μ. Theparticle motionbecomesstochasticasaresultofthecyclotroninteractionoffastionswiththehelical ripplesof stellaratormagneticfield.Theappropriateconsiderationfor tokamakshasbeen donein[8].Thecontributionofthestochastictransporttothelossoffastionsinthe stellaratorexperimentsisdiscussed.

2. CYCLOTRONINTERACTIONOFFASTIONSWITHHELICALRIPPLES

Thecyclotroninteractionofafastionwitharippledmagneticfieldmaybedescribed bytheequations

$$\dot{\mu}=V_{\perp}\left(V^{2}+V_{\|}^{2}\right)/(B\rho)\sin\alpha,\quad \dot{\alpha}=-\omega_{Bi}\ .(1)$$

Here $\mu=V_{\perp}^{2}/B$ isthemagneticmoment, αthegyro-phaseand $V_{\perp}$ theperpendicular componentoftheparticlevelocity, ω_{Bi}thecyclotronfrequencyand ρ isthecurvatureradius ofthemagneticlineofforcedeterminedas

$$\rho^{-1}=\left|\nabla\left(P+B^{2}/2\right)-\mathbf{B}(\boldsymbol{\tau}_{0}\cdot\nabla)B\right|/B^{2},\quad \boldsymbol{\tau}_{0}=\mathbf{B}/B\ ,(2)$$

*P*isaplasmapressure.Forthecaseofaslowlyvaryingmagnitudeof $\dot{\mu}$,i.e.for $M\equiv V_{\perp}\left(V^{2}+V_{\|}^{2}\right)/(B\rho)<<\mu\omega_{Bi}$, Eq.(1)yieldstheregulargyro-oscillationsofmagnetic moment $\tilde{\mu}=\mu-\bar{\mu}$ givenby

$$\tilde{\mu}/\mu=M/(\mu\omega_{B})\cos\alpha\cong(\rho_{L}/\rho)(V/V_{\perp})\cos\alpha\ .(3)$$

with $V_{\|}$ thelongitudinalcomponentoftheparticlevelocityand ρ_L=V $/\omega_{Bi}$ thegyro-radius and $\bar{\mu}$ thegyro-averagedmagneticmoment.Inthiscase gyro-averagingoftherightsideof (3)wouldyieldzeroimplyingtheconservationof $\bar{\mu}$ .Typicaloscillationsof μ fora circulating38 *keV*deuteroninCHSareshowninFig.1.Duetothehelicalnatureof stellarator magneticfieldthemagnitude *M*maycontaintheoscillatoryterms ~$\exp[i(n\vartheta-N\varphi)]$withrather high toroidalmodenumbers $N>>m$,where *m*isthe toroidalperiodnumberof stellarator, *n*is poloidalmodenumber.Theyappearmainlyduetothedependenceof *M*onthecurvature radius $\rho(\vartheta,\varphi)$.Itshouldbepointedoutherethatthecurvatureradiusmaycontainhelical harmonics with high N even in the case when $B(\vartheta,\varphi)$ does not. It is confirmed by the profiles of the helical harmonics (4,16) and (6,24) of the curvature radius expansion ($\rho_0/\rho = \sum \kappa_{nN} \cos(n\vartheta - N\varphi),\ \kappa_{00} \equiv 1$) for CHS with R=92 *cm* shown in Fig.1. For comparison in Fig.2 also the radial profiles of the corresponding harmonics of magnetic field are shown. The high-N helical modes of $\rho(\vartheta,\varphi)$ will result in the cyclotron interaction of the particle gyration and fast oscillations of M. To estimate the effect of non-conservation of μ caused by this interaction we expand M in a series of helical harmonics

$$M(\vartheta,\varphi) = \sum M_{nN} \exp\left[i(n\vartheta - N\varphi)\right]. \quad (4)$$

and rewrite equation for $\dot{\mu}$ as

$$\dot{\mu} = -0.5i \sum M_{nN}\left[\exp\left(i\psi_{nN}^{+}\right) - \exp\left(i\psi_{nN}^{-}\right)\right], \quad \psi_{nN}^{\pm} = n\vartheta - N\varphi \pm \alpha \ . \quad (5)$$

From the above equation it follows that in the presence of high-N harmonics corresponding to the cyclotron resonance $\dot{\psi}_{lN}^{\pm} \equiv n\dot{\vartheta} - N\dot{\varphi} \pm \dot{\alpha} = 0$, $\overline{\mu}$ will not be conserved. In the lowest order of drift approximation the resonant condition is given by

$$V_{\|}(B) = R_0 \omega_{B_0} \big/ (N - \iota n) \equiv V_{\|r}(r) . \quad (6)$$

Wherefrom, it follows that the cyclotron interaction is only possible for fast ions which satisfy

$$V > R_0 \omega_{B_0} \big/ (N - \iota n), \quad \rho_L > R_0 \big/ (N - \iota n) . \quad (7)$$

Next we consider the important feature of the local character of the cyclotron resonance, i.e. that it is possible only in the vicinity of the toroidal position $\varphi = \varphi_r(r, V, \mu)$ along the orbit satisfying the condition $V_{\|}(B(\varphi_r)) = V_{\|r}(r)$ (see Fig.5). In a conventional stellarator configuration ($\iota\varepsilon_t << m\varepsilon_h$) the resonant region in toroidal coordinate $\delta\varphi$ (corresponding with $\delta\psi_{nN}^{\sigma} = \psi_{nN}^{\sigma}(\varphi_r) - \psi_{nN}^{\sigma}(\varphi) \le 1$), that contributes mainly to the variation of $\overline{\mu}$, is small $\delta\varphi \approx \left|(\dot{\alpha}/\dot{\varphi})_r'\right|^{-1/2} \propto V_{\|} \Big/ \left(V\sqrt{\varepsilon_h (N - \iota n)(m - \iota n)}\right) << 1$, here $(\ldots)'$ denotes the derivative with respect to φ. This confirms the local character of cyclotron resonance. With the help of Eq. (5), in the lowest order of the stationary phase method, we arrive for the variation $\Delta\mu_r$ of magnetic moment, caused by the cyclotron resonance at $\varphi = \varphi_r$, at the expression

$$\Delta\mu_r \cong \Delta\mu \sin\left[\psi_{lN}^{\sigma}(\varphi_r) + \operatorname{sgn}\psi_{lN}''^{\sigma}(\varphi_r)\pi/4\right], \quad \Delta\mu = \sqrt{\pi \big/ \left(|N - \iota l|\,|\mu B'(\varphi_r)|\right)}\, M_{lN} R_0 . \quad (8)$$

Then the evolution of magnetic moment may be described by the following mapping

$$\mu_{n+1} - \mu_n = \Delta\left(\mu_n, \varphi_r^n(\mu_n)\right)\cos\Psi_n, \quad \Psi_{n+1} - \Psi_n = (-1)^n G(\mu_{n+1}) + P(\mu_{n+1}) . \quad (9)$$

Here $\Psi = \psi_{lN}^{\sigma}(\varphi_r) + \operatorname{sgn}\psi_{lN}''^{\sigma}(\varphi_r)\pi/4$; μ_n and Ψ_n are the values of μ and Ψ at some pass through the resonant point φ_r^n; μ_{n+1}, Ψ_{n+1} are the values of the same variables during the pass of point φ_r^{n+1}; $G(\mu) = (N - l\iota)\int_{\Delta\varphi} d\varphi \left|V_{\|r}/V_{\|} - 1\right|$; $P(\mu) = -\rho_L \mu R N/(2r\iota V)\int_{\Delta\varphi} d\varphi V_{\|}^{-1}\,\partial B/\partial r$.

Mapping (9) is similar to the one, which describes the stochastic diffusion induced by cyclotron interaction of fast particles with TF ripples in tokamaks [3]. It differs, however, from tokamak mapping by the existence of a several resonant points along the orbit (Fig.5), which form the effective resonant regions (Fig.6). The next stellarator peculiarity is the existence of resonant levels with different poloidal mode numbers (Fig.6) that decreases the distance between the neighboring resonant levels by a factor m/ι. Therefore as a stochasticity criterion we can use $\left|\Delta\mu\,\partial G/\partial\mu\right| > \iota/m$ which yields the following qualitative evaluation of stochasticity threshold for the helical harmonics of ρ^{-1}

$$\kappa_{nN} > \kappa_{cr} \approx \pi^{-1}\rho_0 R\iota\sqrt{\varepsilon_h (m-\iota l)^{-1}(N-\iota n)^{-5}}\,\rho_L^{-2} \propto \pi^{-1}\iota\sqrt{\varepsilon_h (m-\iota l)^{-1}(N-\iota n)^{-1}} \quad (10)$$

It follows from (10) that for 38 keV untrapped deuterons resonating with N=3m=24 harmonics in CHS (l=2, m=8, R =92 cm, B =0.9 T) at r/a=0.8($\iota \approx 0.7, \varepsilon_h \approx 0.17$) the critical value of $\kappa_{n,24}$ is less then 1%. Therefore, at least at the CHS plasma periphery, where $\kappa_{6,24}(r \approx a) \approx 0.3\%$, one may expect the stochastic diffusion of fast co-circulating ions with $D_{\mu\mu} \propto (\Delta\mu)^2 \omega_b$, here $\omega_b = \iota V_{\parallel} / R$ is the bounce frequency. This diffusion corresponds to the following effective pitch angle scattering in normalized magnetic moment $\lambda = \mu B_0 / V^2$

$$D_{\lambda\lambda} \cong (R/\rho_{curv})^2 \varepsilon_h^{-1} (N-\iota n)^{-1}(m-\iota l)^{-1} \kappa_{lN}^2 \omega_b \propto (\iota R/\pi\rho_{curv})^2 (N-\iota n)^{-2}(m-\iota l)^{-2} \omega_b . \quad (11)$$

For the stochastic diffusion induced by cyclotron interaction of deuterons with N=24 harmonics (11) yields $D_{\lambda\lambda}$ ~2÷50 s^{-1} that significantly exceeds the typical collisional pitch-angle scattering rates $\nu_\perp$~10^{-1} s^{-1}. This high stochastic pitch-angle scattering rate yields rather low confinement time τ_n~$1/D_{\lambda\lambda}$~20÷500 ms and may be considered as one of the mechanisms responsible for loss of NBI ions on CHS in the case of tangential injection [4-7].

Let us estimate now the rate of stochastic radial diffusion of toroidally trapped fast particles in stellarator due to non-conservation of the longitudinal adiabatic invariant [2]. The corresponding stochasticity criterion may be written as follows

$$\Gamma l \rho_L \sqrt{\varepsilon_h} \big/ (2 r \iota \varepsilon_t) > 1, \quad (12)$$

where $\Gamma = \ln[16 e (m-\iota l)\varepsilon_h / (\pi\iota\varepsilon_t |\sin\vartheta|)] \geq 5 \div 6$. Thus one may expect that toroidally trapped fast ions with ρ_L/r ≥0.1 ÷0.2 will diffuse with a high radial diffusion rate $D \cong \Gamma^2 \rho_L^2 \iota V \big/ \left[R(m-\iota l)^2 \sqrt{\varepsilon_h}\right]$ resulting in the low confinement time of them

$$\tau_n^t \approx r^2 R (m-\iota l)^2 \sqrt{\varepsilon_h} \big/ \left(\Gamma^2 \rho_L^2 \iota V\right). \quad (13)$$

For the typical CHS parameters we obtain τ_n^t ~10÷100 μs. Note that due to the high probability of the collisionless transformation of localized orbit into toroidally trapped and vice versa the fast loss of toroidally trapped ions may significantly influence the confinement of helically trapped ions in the case of perpendicular injection.

3. SUMMARY

The invariance breakdown of the magnetic moment due to the cyclotron interaction of fast ions with a rippled magnetic field in a stellarator is shown to occur in the case of a relatively large ratio of gyro-to flux surface radius, $\rho_L/r \geq$ 0.1÷0.2. Non-adiabaticity of μ is mainly due to the interaction with helical harmonics of curvature radius with high toroidal numbers N. If the magnitudes of these high-N harmonics exceed some critical values the transition to the stochasticity resulting in a collisionless pitch-angle scattering of fast ions takes place. This collisionless scattering may even exceed the collisional pitch-angle scattering rate and may be important for explanation the enhanced loss of tangentially injected deuterons on CHS in the case of low magnetic field [4].

The non-conservation of the longitudinal adiabatic invariant, resulting in stochastic diffusion of toroidally trapped fast ions, is expected significantly to weaken the confinement of perpendicular injected ions with $\rho_L/r \geq$ 0.1÷0.2: This may be important for interpreting fast ion loss measurements in present-day stellarators [4-7]. We note, however, that τ_n^t strongly depends also on the number of field periods m; hence, for instance, the ratio of confinement times of toroidally trapped NBI ions in Heliotron-E(m=19) [8] and CHS is rather high, $\tau_{nHE}^t / \tau_{nCHS}^t \geq 10^2$ (in spite that $\rho_{LCHS}/\rho_{LHE} \approx 2$). This is in qualitative agreement with the better confinement of ions injected perpendicularly into Heliotron-E. Note that the weakening of non-adiabatic effects with decreasing ρ_L/r should improve the confinement of high-energy ions in reactor-size machines.

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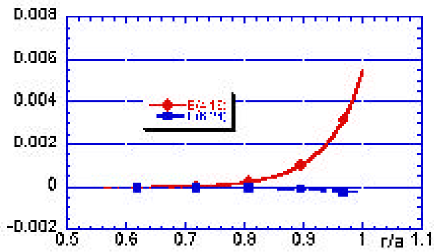

Fig.1 Profiles of n=4, N=16 and n=6, N=24 magnetic field harmonics in CHS (R=92)

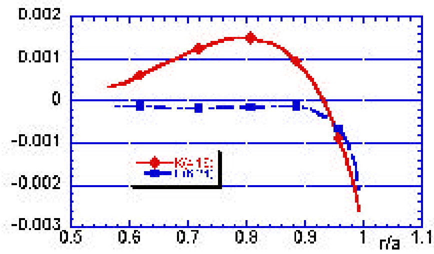

Fig.2 Profiles of n=4, N=16 and n=6, N=24 curvature harmonics in CHS (R=92cm)

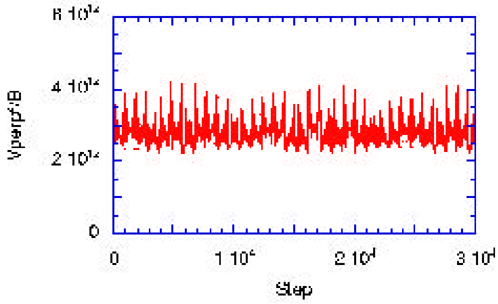

Fig.3 Gyrooscillations of μ for 38 *keV* deuteron with initial pitch angle 132.5 °in CHS

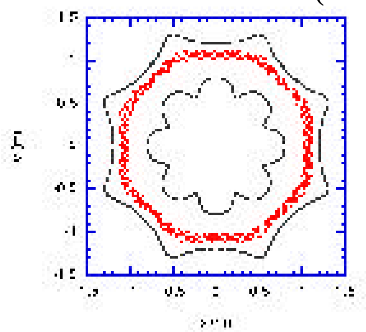

Fig.4 Toroidal projection of orbit of 38 *keV* deuteron with initial pitch angle 132.5 °in CHS

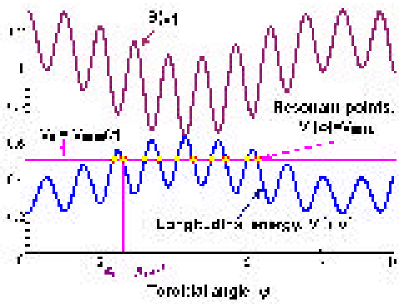

Fig.5 Resonant points in toroidal angle

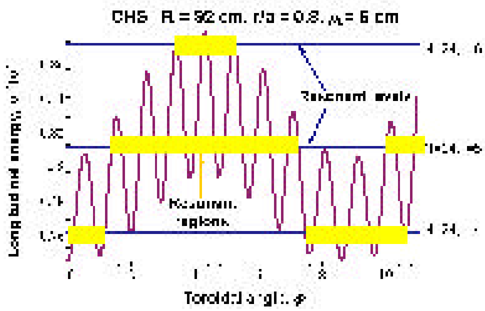

Fig.6 Resonant regions for 38 *keV* deuterons in CHS