STOCHASTIC TRANSPORT OFF FASTIONS IN STELLARARTORS


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1. INTRODUCTION

Stochastic collisionless transport is known to contribute significantly to the loss of energetic ions in toroidal magnetic confinement devices even in the case of a tokamak with weak axial symmetry of the magnetic field [1]. Helical ripples of a stellarator magnetic field considerably complicate the fast particle motion as compared to that in the tokamak, and may cause the stochastic behaviour and stochasticity induced collisionless loss mechanisms of high-energy particles in the helical confinement systems as well. Thus, the motion of energetic toroidally trapped particles with the toroidal precession being in resonance with the helical ripple perturbations becomes stochastic due to the non-conservation of the longitudinal adiabatic invariant \( J \).

The main purpose of the present paper is the study of the stochastic transport of fast ions in stellarators induced by the non-conservation of the magnetic moment \( \mu \). The particle motion becomes stochastic as a result of the cyclotron interaction of fast ions with the helical ripples of the stellarator magnetic field. The appropriate consideration for tokamaks has been done in [8]. The contribution of the stochastic transport of fast ions in the stellarator experiments is discussed.

2. CYCLOTRON INTERACTION OFF FASTIONS WITH HELICAL RIPPLES

The cyclotron interaction of fast ions with helical ripples of the magnetic field may be described by the equations

\[
\frac{\dot{V}}{V} = \frac{V^2 + V^2_{\perp}}{(B\rho)} \sin \alpha, \quad \dot{\alpha} = -\omega_B. \tag{1}
\]

Here \( \mu = V^2_{\perp}/B \) is the magnetic moment, \( \alpha \) the gyro-phase, \( V_{\perp} \) the perpendicular component of the particle velocity, \( \omega_B \) the cyclotron frequency, and \( \rho \) the curvature radius of the magnetic line of force determined as

\[
\rho^{-1} = \nabla \cdot \left( \frac{P + B^2/2}{B^2} - B\tau_0 \cdot \nabla \right) B, \quad \tau_0 = B/B. \tag{2}
\]

\( P \) is the plasma pressure. For the case of a slowly varying magnitude of \( \mu \), i.e., \( \mu = M/(B\rho) \ll \mu \omega_B \), Eq. (1) yields the regular gyro-oscillations of magnetic moment \( \tilde{\mu} = \mu - \mu_B \) given by

\[
\tilde{\mu}/\mu = M/(\mu \omega_B) \cos \alpha \approx (\rho_{\perp}/\rho)(V/V_{\perp}) \cos \alpha. \tag{3}
\]

with \( V_{\perp} \) the longitudinal component of the particle velocity and \( \rho_{\perp} = V_{\perp} / \omega_B \) the gyro-radius and \( \mu_B \) the gyro-averaged magnetic moment. In this case, gyro-averaging of the right side of Eq. (3) would yield zero implying the conservation of \( \mu \). Typical oscillations of \( \mu \) for circulating \( 38 \) keV deuterons in CHS are shown in Fig. 1. Due to the helical nature of the stellarator magnetic field, the magnitude \( M \) may contain oscillatory terms \( \exp(i(n \varphi - N \varphi)) \) with rather high toroidal mode numbers \( N \gg m \), where \( m \) is the toroidal period number of the stellarator, \( n \) is the poloidal mode number. They appear mainly due to the dependence of \( M \) on the curvature radius \( \rho(\vartheta, \phi) \). It should be pointed out that the curvature radius may contain helical...
harmonics with high Neven then the case when \( B(\vartheta, \phi) \) does not. It is confirmed by the profiles of the helical harmonics (4, 16) and (6, 24) of the curvature radius expansion

\[
\rho / \rho = \sum \kappa_{n} \cos(n \vartheta - N \phi), \kappa_{00} = 1
\]

for CHS with \( R = 92 \ cm \) shown in Fig. 1. For comparison in Fig. 2 the radial profiles of the corresponding harmonics of magnetic field are shown. The high-

harmonics of \( \vartheta, \phi \) will result in the cyclotron interaction of the particle gyration and fast oscillations of \( M \). To estimate the effect of non-conservation of \( \mu \) caused by this interaction we expand \( M \) in a series of helical harmonics

\[
M(\vartheta, \phi) = \sum M_{m} \exp [(n \vartheta - N \phi)]
\]

and rewrite the equation for \( \mu \) as

\[
\dot{\mu} = -0.5i \sum M_{m} \left[ \exp(i \psi_{n^+}^m) - \exp(i \psi_{n^-}^m) \right], \quad \psi_{n}^\pm = n \vartheta - N \phi \pm \alpha. \tag{5}
\]

From the above equation it follows that in the presence of high-

harmonics corresponding to the cyclotron resonance \( \psi_{n}^\pm = n \vartheta - N \phi \pm \alpha = 0 \), \( \mu \) will not be conserved. In the lowest order of drift approximation the resonant condition is given by

\[
V_{\mu}(B) = R_{\vartheta} \omega_{\vartheta} / (N - m) \equiv V_{\mu}^{\prime}(r). \tag{6}
\]

Wherefrom, it follows that the cyclotron interaction is only possible for fast ions which satisfy

\[
V > R_{\vartheta} \omega_{\vartheta} / (N - m), \quad \rho_{L} > R_{\vartheta} / (N - m). \tag{7}
\]

Next, we consider the important feature of the local character of the cyclotron resonance, i.e.

that it is possible only in the vicinity of the resonant position \( \varphi = \varphi_{r}(r, V, \mu) \) along the orbit satisfying the condition

\[
V_{\mu}(B(\varphi)) = V_{\mu}^{\prime}(r) \quad \text{(see Fig. 5)}.
\]

In a conventional stellarator configuration \((\epsilon, \epsilon_{r} \leq m \epsilon_{h})\) the resonant region is a toroidal coordinate \( \delta \varphi \) corresponding with the cyclotron resonance \( \psi_{n}^\pm = \psi_{n}^{\alpha}(\varphi) - \psi_{n}^{\alpha}(\varphi) \leq 1 \), that contributes mainly to the variation of \( \mu \) inside small

\[
\delta \varphi = \left( \alpha / \varphi_{r} \right)^{1 / 2} \sim V_{\mu}^{\prime} / V_{\mu} \left( \epsilon_{r}(N - m)(m - m) \right) \ll 1. \text{(here…)}
\]

\( \delta \varphi \) denotes the derivative with respect to \( \varphi \). This confirms the local character of the cyclotron resonance. With the help of Eq. (5), in the lowest order of the stationary phase method, we arrive for the variation of the magnetic moment caused by the cyclotron resonance at

\[
\Delta \mu = \Delta \mu_{i} \Delta \mu_{s} = \Delta \mu_{s} \sin \psi_{n}^{\alpha}(\varphi) + \text{sgn} \psi_{n}^{\alpha}(\varphi) \pi / 4.
\]

Then the evolution of the magnetic moment may be described by the following mapping

\[
\mu_{n+1} - \mu_{n} = \Delta \mu_{i} \cos \Psi_{n}, \quad \Psi_{n+1} = \Psi_{n} - (\pi / 4) G(\mu_{n+1}) + P(\mu_{n+1}). \tag{8}
\]

Here \( \Psi = \psi_{n}^{\alpha}(\varphi) + \text{sgn} \psi_{n}^{\alpha}(\varphi) \pi / 4 \); \( \mu_{n} \) and \( \Psi_{n} \) are the values of \( \mu \) and \( \Psi \) at some point through the resonant point \( \varphi_{r}^{n+1} \); \( \mu_{n+1} \), \( \Psi_{n+1} \) are the values of the same variables during the pass of point \( \varphi_{r}^{n+1} \); \( G(\mu) = (N - m) \int_{\varphi} d \varphi V_{\mu}^{\prime 1} \delta B / \delta \varphi \); \( P(\mu) = - \rho_{L} \mu R / (2 r_{V}) \int_{\varphi} d \varphi V_{\mu}^{\prime 1} \delta B / \delta \varphi \).

Mapping (9) is similar to the one that describes the stochastic diffusion induced by cyclotron interaction of fast particles with TF ripples in tokamaks [3]. It differs, however, from tokamak mapping by the existence of several resonant points along the orbit (Fig. 5), which form the effective resonant regions (Fig. 6). Then the next stellarator peculiarity is the existence of resonant nodes with different poloidal mode numbers (Fig. 6) that decrease the distance between the neighboring resonant levels by a factor \( m / k \). Therefore, as a stochasticity criterion we can use

\[
|\Delta \mu \partial G / \partial \mu| > 1 / m \]

which yields the following qualitative evaluation of the stochastic threshold for the helical harmonics of \( \rho_{\mu} \).
The invariance breakdown of the magnetic moment due to the cyclotron interaction of fast ions with a rippled magnetic field in a stellarator shows two occurrence cases of relatively larger ratio of gyro- to flux surface radius, $\rho_L/\rho \geq 0.1 \pm 0.2$. Non-adiabaticity of $\mu$ is mainly due to the interaction with helical harmonics of curvature radius with high toroidal numbers $N$. If the magnitudes of these high harmonics exceed some critical values, the transition to the stochasticity resulting in collisionless pitch-angle scattering of fast ions takes place. This collisionless scattering may even exceed the collisional pitch-angle scattering rate and may be important for explaining the enhanced loss of tangentially injected deuterons in the case of CHS in the case of non-stochastic angular diffusion. 

\[ \kappa_n > \kappa_{cr} = \pi^2 \rho_0 R \sqrt{e_n} (m-u)^{-1} (N-u)^{-3} \rho_L^{-2} \propto \pi^2 \sqrt{\frac{e_n}{m-u}} (N-u)^{-3}. \]

It follows from (10) that for $38 \text{keV}$ untrapped deuterons resonating with $N=3m=24$ harmonics in CHS $(l=2, m=8, R=92 \text{ cm}, B=0.9 \text{ T})$ at $r/a=0.8 (\theta=0.7, \varepsilon_n=0.17)$ the critical value of $\kappa_{cr}$ is less than $1\%$. Therefore, at least at the CHS plasmaperiphery, where $\kappa_{cr}(r=a)=0.3\%$, one may expect the stochasticity criterion may be written as follows:

\[ D_{\mu\nu} \propto (\Delta \mu)^2 \omega_b \text{, here } \omega_b = \nu/\rho \text{, } R \text{ is the bounce frequency. This diffusion corresponds to the following effective pitch-angle scattering in a normalized magnetic moment: } \lambda = \mu B_0/V^2. \]

For the stochastic diffusion induced by cyclotron interaction of deuterons with $N=24$ harmonics (11) yields $D_{\mu\nu} \sim 2\pm 50 \text{ s}^{-1}$ that significantly exceeds the typical collisional pitch-angle scattering rate $v_L \sim 10^{-1} \text{ s}^{-1}$. This stochastic pitch-angle scattering rate yields a rather low confinement time $\tau_n \sim 1/D_{\mu\nu} \sim 20\pm 500 \text{ ms}$ and may be considered as one of the mechanisms responsible for the loss of NB ions in the case of fast injection [4-7].

Let us estimate now the rate of stochastic radial diffusion of toroidally trapped fast particles in the case of tangential injection of NB ions in the case of non-stochastic angular diffusion of fast ions with $\rho_L/\rho \geq 0.1 \pm 0.2$. This may be important for interpreting fast ion loss measurements in present-day stellarators [4-7]. We note, however, that $\tau_n$ strongly depends on the number of field periods $m$, for instance, the ratio of confinement times of toroidally trapped NB ions in Heliotron-E $(m=19)$ [8] and CHS is rather high, $\tau_{n\text{HE}}/\tau_{n\text{CHS}} \geq 10^2$ (inspite that $\rho_{L\text{CHS}}/\rho_{L\text{HE}}=2$). This is a qualitative agreement with the better.
confinement of ions injected perpendicularly into Heliotron-E. Note that the weakening of non-adiabatic effects with decreasing $\rho/r$ should improve the confinement of high-energy ions in reactor-size machines.

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