27th EPS Conference on Contr. Fusion and Plasma Phys. Budapest, 12-16 June 2000 ECA Vol. **24B** (2000) 280 -283

Rates of thermonuclear reactions in dense plasmas

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Abstract The thermonuclear reaction rates in a dense plasma are evaluated on account of both dynamic screening and plasma fluctuations, with the result that the reaction rates are slowed down with respect to their vacuum values, in contrast with the enhancement produced by static screening and in the absence of plasma fluctuations. The slowing down relevant to the nuclear reactions in the Sun is evaluated in view of the solar neutrino problem.

Introduction. It is well–known that the rates R_{ij} of the thermonuclear reactions in plasmas are enhanced over their vacuum values, $R_{ij}^{(0)}$, due to the **static** screening of the reacting nuclei (ions) i and j, the effects of plasma fluctuations being disregarded. (Salpeter's enhancement [1]). More specifically, in the weak screening limit, for which the screening potential is the Debye–Hückel potential, one can write $R_{ij} = \left(1+\Lambda_{ij}\right)R_{ij}^{(0)}$, with $\left|\Lambda_{ij}\right| << 1$, and Salpeter's enhancement is given by [2]

$$\Lambda_{ij} = -\frac{1}{T}\left[-\frac{\left(Z_i+Z_j\right)^2 e^2}{2d}+\frac{\left(Z_i^2+Z_j^2\right)e^2}{2d}\right]=\frac{Z_i Z_j e^2}{Td}\equiv\Lambda_{ij}^S \qquad (1)$$

the quantity within the square brackets being the difference between the free energy of the screened (stationary) ions (charges $Z_i e$ and $Z_j e$) at zero separation, $\left[-\left(Z_i+Z_j\right)^2 e^2 \big/ 2d\right]$, that equals the (Helmholtz) free energy of a single screened ion of charge $\left(Z_i+Z_j\right)e$, and the corresponding free energy at infinite ion separation. In (1), $d^{-1} = \left[4\pi n_e e^2\left(1+Z^*\right)\big/T\right]^{1/2}$ is the reciprocal Debye length, where $Z^* \equiv \sum_i\left(Z_i^2 n_i\right)\big/n_e$ is the mean charge number of the relatively light ions that contribute to the static screening; $n_e\left(=\sum_i Z_i n_i\right)$ is the electron number density and T is the plasma temperature (in energy units).

Besides neglecting the effects of plasma fluctuations, the static screening result (1) is subject to the condition that the motion of the screened ions is slow compared to the motion of the screening charges. On noting that the energies of the screened, reacting nuclei are about the Gamov energy, that is, in general, higher than the corresponding thermal energy, it is needed to re–calculate the reaction rates in a plasma on the basis of the kinetic plasma theory, which, in particular, permits to account for the collective plasma effects, namely, dynamic screening [3,4] along with plasma fluctuations [5].

The nuclear reaction rate in plasmas. The nuclear reaction rate R_{ij} is given by

$$R_{ij} = \frac{1}{1+\delta_{ij}}\int\frac{d\mathbf{p}\,d\mathbf{p}'}{(2\pi)^6}\left\langle\overline{w}_{ij}(\mathbf{p},\mathbf{p}')\,f_i(\mathbf{p})\,f_j(\mathbf{p}')\right\rangle \qquad (2)$$

where $\overline{w}_{ij}(\mathbf{p},\mathbf{p}') = w_{ij}(\mathbf{p},\mathbf{p}') + w_{ji}(\mathbf{p}',\mathbf{p})$ is the (total) tunneling probability; $f(\mathbf{p}) \equiv f(\mathbf{p},\mathbf{r},t)$ is the momentum distribution function $\left(\int \frac{d\mathbf{p}}{(2\pi)^3} f_i(\mathbf{p}) = n_i\right)$, and $\langle \ldots \rangle$ denotes the average with respect to fluctuations. To evaluate (2), one proceeds along the same lines as for the derivation of the Balescu–Lenard collision integral, the plasma effects on the nuclear reactions being specifically accounted for as follows:

i) the tunneling probability $\overline{w}_{ij}$ is expanded about its vacuum value $w_{ij}^{(0)}$ in powers of the (polarization) fluctuating potential $\delta\phi$, the corresponding 1st and 2nd order corrections being evaluated from the quasi–classical solution of Schrödinger equation [5b]. The resulting contribution to (the integrand of) (2) is

$$\left[\frac{(Z_i+Z_j)e}{T}\left(\langle\delta\phi\delta f_i\rangle\Phi_j + i \leftrightarrow j\right) + \frac{(Z_i+Z_j)^2 e^2}{2T^2}\left\langle(\delta\phi)^2\right\rangle\Phi_i\Phi_j\right] w_{ij}^{(0)}(E_r) \tag{3}$$

$$= \left[-(Z_i+Z_j)^2 + \frac{1}{2}(Z_i+Z_j)^2\right]\frac{e^2}{Td} w_{ij}^{(0)}(E_r)\Phi_i\Phi_j$$

the equality following from making use of the (standard) results of the kinetic theory of (thermal) plasma fluctuations, namely, $\langle\delta\phi\delta f_i\rangle = -(Z_i e/d)\Phi_i(\mathbf{p})$ and $\left\langle(\delta\phi)^2\right\rangle = (T/d)$, ($w_{ij}^{(0)}(E_r) \equiv \overline{w}_{ij}^{(0)}(\mathbf{p},\mathbf{p}')$, with E_r the relative kinetic energy of the two reacting nuclei; Φ and δf are, respectively, the average and fluctuating part of the distribution function). Note that the contribution to R_{ij} from the 2nd term within the square brackets of (3) is just the 1st term within the square brackets of (1).

ii) the contribution to (2) connected with the zeroth order (in $\delta\phi$) tunneling probability $\overline{w}_{ij}^{(0)}$ is simply

$$\overline{w}_{ij}^{(0)}\langle f_i f_j\rangle = \overline{w}_{ij}^{(0)}\left[\Phi_i\Phi_j + \langle\delta f_i\delta f_j\rangle\right] = \overline{w}_{ij}^{(0)}\Phi_i\Phi_j + \frac{Z_iZ_je^2}{Td} w_{ij}^{(0)}(E_r)\Phi_i\Phi_j \tag{4a}$$

with $\langle\delta f_i\delta f_j\rangle = \left(Z_iZ_je^2/Td\right)\Phi_i\Phi_j$, in accordance with the kinetic theory of (thermal) plasma fluctuations. As for the 1st term on the r.h.s. of (4a), it has to be evaluated with account of the slow–time variation of both Φ_i and Φ_j due to the nuclear reactions themselves (note that, instead, in the derivation of the Balescu–Lenard collision integral, the time scale of variation of Φ is taken to be much longer than that characteristic of the plasma fluctuations, so that Φ is treated as time–independent). More specifically, with reference to the kinetic equation $\frac{\partial\Phi}{\partial t} = Ze\frac{\partial}{\partial\mathbf{p}}\cdot\langle(\nabla\delta\phi)\delta f\rangle$, the Fourier transform of δf, in addition to the term of the standard kinetic theory, comprises a term proportional to $\left(\frac{\partial\delta\phi_{\mathbf{k},\omega}}{\partial\omega}\frac{\partial\Phi}{\partial t}\right)$, the effect of which amounts to the renormalization of the distribution function

$$\Phi_i(\mathbf{p}) \to \Phi_i(\mathbf{p}) + \frac{1}{T}\left[\frac{(Z_ie)^2}{d} + I_i^{(f)}(\mathbf{v})\right]\Phi_i(\mathbf{p}) \tag{5a}$$

where

$$I_i^{(f)}(\mathbf{v}) = \frac{(Z_i e)^2}{2\pi^2}\int\frac{d\mathbf{k}}{k^2}\left[\omega^2\frac{\partial}{\partial\omega}\frac{1}{\omega}\left(1-\frac{1}{\varepsilon_{\mathbf{k},\omega}}\right)+\frac{T}{m_i v^2}\frac{\partial}{\partial\omega}\omega^3\frac{\partial}{\partial\omega}\frac{1}{\omega}\left(\frac{1}{\varepsilon_{\mathbf{k},\omega}}-\frac{1}{\varepsilon_{\mathbf{k},0}}\right)\right]_{\omega=\mathbf{k}\cdot\mathbf{v}} \quad (5b)$$

which describes the effects of dynamic screening on account of plasma fluctuations, through the (electrostatic) dielectric function $\varepsilon_{\mathbf{k},\omega}$.

As for the physical meaning of the renormalization (5a), on expressing the reciprocal of the Debye length in terms of the static screening function $\left(1/\varepsilon_{\mathbf{k},0}\right)$, i.e., $1/d = \int d\mathbf{k}\left(1-1/\varepsilon_{\mathbf{k},0}\right)/\left(2\pi^2 k^2\right)$, one can re–write the quantity within the square brackets in (5a) as

$$[\ldots] = -\frac{(Z_i e)^2}{2\pi^2}\int\frac{d\mathbf{k}}{k^2}\left[\omega^2\frac{\partial}{\partial\omega}\frac{1}{\omega}\left(\frac{1}{\varepsilon_{\mathbf{k},\omega}}-\frac{1}{\varepsilon_{\mathbf{k},0}}\right)\right]_{\omega=\mathbf{k}\cdot\mathbf{v}} = 2E_{Z_i^2}(\mathbf{v}) \quad (5c)$$

the contribution (5b) being taken to lowest order in $\left(T/m_i v^2\right)$ (<<1, the reacting nuclei being superthermal). Thus, by renormalizing the distribution function, as in (5a), one accounts for the self–energy, $E_{Z_i^2}(\mathbf{v})$, of the screened (dressed) plasma particles, as opposed to bare particles. One should note that the (negative) self–energy (5c) is a function of the particle velocity.

The renormalization (5a) applied to the 1st term of (4a) yields

$$\overline{w}_{ij}^{(0)}\Phi_i\Phi_j \to \left[1+\frac{1}{2T}\left(\frac{\left(Z_i^2+Z_j^2\right)e^2}{d}+I_i^{(f)}(\mathbf{v})+I_j^{(f)}(\mathbf{v}')\right)\right]w_{ij}^{(0)}(E_r)\Phi_i\Phi_j \quad (4b)$$

On adding up the four terms connected with the static screening, i.e., the contribution (3), the last term on the r.h.s. of (4a) and the 2nd term within the square brackets of (4b), one gets a complete cancellation, i.e., **the net effect of the static screening is zero**! Such a result is in sharp contrast with the enhancement (1).

The only non–zero contribution to the reaction rate (2), relative to its vacuum value, $R_{ij}^{(0)}$, is thus connected with the two $I^{(f)}$–terms of (4b), with the result that $R_{ij} = \left(1+\Lambda_{ij}\right)R_{ij}^{(0)}$, with, to lowest order in $\left(T/m_i v^2\right)$,

$$\Lambda_{ij} = \frac{e^2}{4\pi^2 T\left(1+\delta_{ij}\right)R_{ij}^{(0)}}\int\frac{d\mathbf{p}\,d\mathbf{p}'}{(2\pi)^6}w_{ij}^{(0)}(E_r)\Phi_i(\mathbf{p})\Phi_j(\mathbf{p}')$$
$$\times\int\frac{d\mathbf{k}}{k^2}\left\{Z_i^2\left[\omega^2\frac{\partial}{\partial\omega}\frac{1}{\omega}\left(1-\frac{1}{\varepsilon_{\mathbf{k},\omega}}\right)\right]_{\omega=\mathbf{k}\cdot\mathbf{v}}+\left(\begin{smallmatrix} i\to j \\ \mathbf{v}\to\mathbf{v}'\end{smallmatrix}\right)\right\} \quad (6)$$

From (6) it appears that i) Λ_{ij} scales as the square of the nuclear charge, as opposed to the $\left(Z_i Z_j\right)$–scaling of the static screening result (1); the greater the nuclear charges, the more important the effect is; ii) the sign of Λ_{ij} is that of the ***k***–integral, which turns out to be negative, so that $\Lambda_{ij} < 0$, i.e., the reaction rates in a plasma are slower than the corresponding ones in vacuo, in contrast with Salpeter's enhancement (1).

Slowing–down of the nuclear reactions for the solar core. The slowing–down (6) of the reaction rates has been calculated for the reactions relevant to the solar core and the numerical results are shown in Table 1.

Table 1. Thermonuclear reactions relevant to the solar plasma. The slowing–down of the thermonuclear reaction rates due to plasma fluctuations, Λ_{ij}, Salpeter's enhancement, $\Lambda_{ij}^S = Z_i Z_j e^2 / Td$, and the factor F_{ij} by which the present reaction rates should be divided.

Reaction	Λ_{ij}	Λ_{ij}^S	F_{ij}
p+p	-0.0510	+0.05	1.106
p+^{2}H	-0.0514	+0.05	1.107
^{3}He+^{3}He	-0.186	+0.20	1.473
^{3}He+^{4}He	-0.190	+0.2	1.481
^{7}Li+p	-0.268	+0.15	1.571
^{7}Be+p	-0.458	+0.2	2.213
^{7}Be+e	-0.446	+0.2	2.166

In respect to the solar neutrino problem, the decrease by about a factor 2 of the rate of both the (^{7}Be, p) and (^{7}Be, e)–reaction is the most relevant result.

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