27th EPS Conference on Contr. Fusion and Plasma Phys. Budapest, 12-16 June 2000 ECA Vol. **24B** (2000) 264 -267

Cross over from relativistic pitch angle scattering to magnetic pinching and X ray conversion efficiency

J. Robiche[1], J.M. Rax[1], I. Kostyukov[1],
E. Besuelle[2], G. Bonnaud[2], F. Walraet[2]
[1] *L.P.M.I Ecole Polytechnique 91128 Palaiseau, France*
[2] *C.E.A D.A.M-D.P.T.A BP12 91680 Bruyeres le Chatel, France*

Introduction

The development of compact, high power, sub-picosecond laser based on chirped pulse amplification has opened a new field of laser plasma interaction. In the terawatt to petawatt regime the electrons quiver velocity becomes relativistic and a whole set of new relativistic non linear processes appears [1]. At the beginning of the decade those processes relevant to the design of advanced accelerators or advanced light sources, such as harmonic generation, wake generation and self focusing have been widely investigated. Recently, besides these fundamental studies on underdense targets, the proposal to use relativistic non linearities to generate a jet of relativistic electrons in order to ignite a thermonuclear target has received a lot of attention. Within the framework of this program two issue are to be addressed: (i) the understanding of the mechanism producing this burst of relativistic electrons in order to optimize the production step; (ii) the precise evaluation of the hot spot size in order to access the potential for ignition. More recently these relativistic electrons burst have been identified as a free energy source for X rays generation through bremsstrahlung in a dense target; again, in order to optimize such a source, a precise evaluation of the electron dynamics is required and moreover the issue of pitch angle scattering becomes of prime importance has this later process is the ultimate source of X ray radiation. The present study addresses the issue of the kinetic relativistic collisional dynamics of energetic electrons. Kinetic theory provides the right framework to address this problem as the electrons-ions interactions can not be reduced to a slowing down and is intrinsically a random process. When a relativistic electron interact with a plasma two type of interactions determine the dynamics: the electrons-electrons interaction and the electrons-ions interaction. Although the first one can be treated as a deterministic slowing down the second one gives rises to pitch angle scattering and determines the size of the energy deposition volume.

That pitch angle scattering is essential to calculate the hot spot size, and the impact of magnetic pinching, can be understood as follow. The slowing down time scale of a relativistic electron is given by : $\tau = 1/4\pi c r_e^2 n_e \log \Lambda_{ee}$, where we have used the classical notation: n_e is the electron density, r_e the classical electron radius and c the velocity of light. The time scale for pitch angle scattering is : $2\tau \log \Lambda_{ee}/Z \log \Lambda_{ei} = 2\tau/Z\Lambda$, where Z is the ion charge state, $\log \Lambda_{ei}$ the Coulomb logarithm for electron ion collisions and $\Lambda \approx 1$

the electron to ion Coulomb logarithm ratio. This scaling means that the commutative Coulomb small angle scattering will turn the electron trajectory by an average angle of $\frac{\pi}{2}$ on a time scale of the order of the slowing down time. Throughout this study we will use the following relativistic collision length,

$$\left[\frac{\lambda}{1mm}\right] = \frac{1}{\log \Lambda_{ee}} \left[\frac{10^{30} m^{-3}}{n_e}\right],$$

as the unit of length and for the clarity rather than the IS system of units we will assume : $e = m = c = \lambda = 1$.

Landau and Beliaev-Budker kinetic operators

Both for fast heating or X ray conversion we have to study the kinetic evolution of a relativistic distribution function , $f(\mathbf{p}, t)$, describing the electron population in terms of the relativistic momentum $\mathbf{p}$ at time t. The electrons-electrons and electrons-ions interactions are described by two Fockker Planck operators $\mathbf{C}_{ee}$ and $\mathbf{C}_{ei}$: $\partial f / \partial t = \mathbf{C}_{ei}[f, F_i] + \mathbf{C}_{ee}[f, F_e]$, where F_e and F_i are the background electrons and ions distribution functions. These collision operators can be expressed in terms of a collision kernel $\mathbf{U}(\mathbf{p}, \mathbf{p}')$ which is nothing but the Fourier transform of the screened Coulomb potential:

$$\mathbf{C}_{ab}[f_a, f_b] = \frac{q_a^2 q_b^2 Log\Lambda_{ab}}{8\pi\varepsilon_0^2} \frac{\partial}{\partial \mathbf{p}} \cdot \int \mathbf{U}(\mathbf{p}, \mathbf{p}') \cdot \left[f_b(\mathbf{p}') \frac{\partial f_a}{\partial \mathbf{p}} - f_a(\mathbf{p}) \frac{\partial f_b}{\partial \mathbf{p}'} \right] d\mathbf{p}'.$$

The classical Landau form of this kernel , $\mathbf{U} = \mathbf{I} - (\mathbf{p} - \mathbf{p}')(\mathbf{p} - \mathbf{p}') / (\mathbf{p} - \mathbf{p}')^2$, becomes the Beliaev-Budker kernel at relativistic velocity [2, 3], γ is the relativistic energy :

$$\mathbf{U} = \frac{(\gamma\gamma' - \mathbf{p}\cdot\mathbf{p}')^2 \left[\left((\gamma\gamma' - \mathbf{p}\cdot\mathbf{p}')^2 - 1 \right) \mathbf{I} - \mathbf{pp} - \mathbf{p}'\mathbf{p}' + (\gamma\gamma' - \mathbf{p}\cdot\mathbf{p}')(\mathbf{pp}' + \mathbf{p}'\mathbf{p}) \right]}{\gamma\gamma' \left((\gamma\gamma' - \mathbf{p}\cdot\mathbf{p}')^2 - 1 \right)^{3/2}}$$

Green function of the Beliaev-Budker Operator

Despite this apparent complexity the Green function of the Beliaev-Budker equation can be calculated [4]. Then this Green function, $G = \exp[\mathbf{C}_{ei} + \mathbf{C}_{ee}]\, t$, can be used to study processes such as the slowing down and the pitch angle scattering of a relativistic electron jet, the size of the hot spot generated through this slowing down and pitch angle scattering, the competition between magnetic pinching and pitch angle scattering and the efficiency of X ray conversion resulting from bremsstrahlung. It turn out that the key parameters describing these processes can be exactly evaluated in compact analytical form so that the scaling with respect to the various control parameters can be understood and the processes optimized.

To give few of these new results let us start with the problem of the hot spot size induced by a relativistic jet. Introducing the spherical momentum coordinates $p_z = p\mu$, $p_x = p\sqrt{1-\mu^2}\cos\varphi$, $p_y = p\sqrt{1-\mu^2}\sin\varphi$. The Green function of the Beliaev-Budker operator can be expanded as a sum over a spherical harmonic set,

$$\begin{aligned} G &= \theta(t-t') \frac{\delta\left[\arctan(p) - p - \arctan(p') + p' - (t-t')\right]}{4\pi\gamma^2} \\ &\times \sum_{l=0}^{l=+\infty} \sum_{m=-l}^{m=+l} Y_l^m(\mu, \varphi) Y_l^{m*}(\mu', \varphi') \left[\frac{p(\gamma'+1)}{p'(\gamma+1)}\right]^{\frac{l(l+1)(Z\Lambda+1)}{2}} \end{aligned}$$

Hot spot size

Based on this propagator a number of interesting results can be derived. First of all the average longitudinal position as a function of the energy,

$$\langle z \rangle (\gamma; \gamma_o) = \left(\frac{\gamma_o + 1}{\gamma_o - 1} \right)^{\frac{Z\Lambda+1}{2}} \int_{\gamma}^{\gamma_o} d\gamma \frac{p^2}{\gamma^2} \left(\frac{\gamma - 1}{\gamma + 1} \right)^{\frac{Z\Lambda+1}{2}} .$$

For example the slowing down length can be easily obtained from this later result when $\gamma = 1$ and $Z = 1$: $\langle z \rangle_o = \lambda \frac{\gamma_0+1}{\gamma_0-1} \left(\gamma_0 - \frac{1}{\gamma_0} - 2 \log \gamma_0 \right)$. Pitch angle scattering shorten this slowing down length by a factor of the order 2, and, moreover, gives rise to a strong longitudinal dispersion of the straggling type,

$$\left\langle z^2 \right\rangle (\gamma; \gamma_o) = \frac{4}{3} \int_{\gamma_o}^{\gamma} d\gamma' \frac{\gamma'^2 - 1}{\gamma'^2} \left(\frac{\gamma' - 1}{\gamma' + 1} \right)^{\frac{Z\Lambda+1}{2}} \times$$

$$\int_{\gamma_o}^{\gamma'} d\gamma'' \frac{\gamma''^2 - 1}{\gamma''^2} \left(\frac{\gamma'' + 1}{\gamma'' - 1} \right)^{\frac{Z\Lambda+1}{2}} + \frac{1}{2} \left(\frac{\gamma_o + 1}{\gamma_o - 1} \right)^{\frac{3(Z\Lambda+1)}{2}} \int_{\gamma_o}^{\gamma'} d\gamma'' \frac{\gamma''^2 - 1}{\gamma''^2} \left(\frac{\gamma'' - 1}{\gamma'' + 1} \right)^{Z\Lambda+1}$$

such that $\langle z^2 \rangle \sim 2 \times \langle z \rangle^2$. Besides this longitudinal dispersion, pitch angle scattering has also a dramatic impact on the transverse size of the hot spot. After some lengthy calculations, involving Clebsch-Gordan calculus, we can express the transverse size of the hot spot as a function of the plasma target parameters as follows:

$$\left\langle r^2 \right\rangle (\gamma; \gamma_o) = \frac{4}{3} \int_{\gamma}^{\gamma_o} d\gamma' \frac{\gamma'^2 - 1}{\gamma'^2} \left(\frac{\gamma' - 1}{\gamma' + 1} \right)^{\frac{Z\Lambda+1}{2}} \times$$

$$\int_{\gamma'}^{\gamma_o} d\gamma'' \frac{\gamma''^2 - 1}{\gamma''^2} \left(\frac{\gamma'' + 1}{\gamma'' - 1} \right)^{\frac{Z\Lambda+1}{2}} - \left(\frac{\gamma_o + 1}{\gamma_o - 1} \right)^{\frac{3(Z\Lambda+1)}{2}} \int_{\gamma'}^{\gamma_o} d\gamma'' \frac{\gamma''^2 - 1}{\gamma''^2} \left(\frac{\gamma'' - 1}{\gamma'' + 1} \right)^{Z\Lambda+1}$$

and a parametric study confirms the dramatic impact of pitch angle scattering: $\langle r^2 \rangle \sim \langle z \rangle^2$. In addition to the three previous results providing the scaling of the size of the hot spot, the Green function of the Beliaev-Budker equation can be used to set up a model describing the competition between pitch angle scattering and magnetic pinching due to the magnetic self field when current neutralization does not occur.

Cross over from magnetic pinching to pitch angle scattering

To set up this model we consider the balance between the magnetic force and the transverse pressure resulting from the electron ion collision : $\mathbf{J} \times \mathbf{B} = \nabla P$. As the magnetic field is self consistently related to the longitudinal electron current through Ampére equation: $\nabla \times \mathbf{B} = \mu_o \mathbf{J}$, it is easy to rewrite this force balance as:

$$\frac{\left\langle p_x^2 + p_y^2 \right\rangle}{\langle\ p_z \rangle} = \frac{I}{I_A}$$

This simple formula describes the criteria of cross over from pitch angle scattering when the current is smaller than the Alfven current I_A to magnetic focusing. Again the Green function of the Beliaev-Budker operator provides a straightforward evaluation of this anisotropy threshold.

$$\frac{\langle p_x^2 + p_y^2 \rangle}{\langle p_z \rangle} = \frac{2}{3} \frac{(\gamma^2 - 1)^{1/2}}{\gamma} \left(\left(\frac{(\gamma - 1)(\gamma_o + 1)}{(\gamma + 1)(\gamma_o - 1)} \right)^{-\frac{(Z\Lambda+1)}{2}} - \left(\frac{(\gamma - 1)(\gamma_o + 1)}{(\gamma + 1)(\gamma_o - 1)} \right)^{Z\Lambda+1} \right)$$

Finally the same type of method a can be applied to access the potential of X ray generation through the use of these laser driven relativistic jet.

X ray conversion efficiency

Given a relativistic electron, with momentum $\mathbf{p}_o$, this electron slow down and heat up the target on a time scale of the order of τ . During this process electrons-ions collisions provide transverse acceleration and bremsstrahlung radiation takes place as a result of pitch angle scattering. The radiated intensity $\frac{dE_x}{d\omega d\Omega_\gamma}$ converted into X rays is given by the formula:

$$\frac{dE_x}{d\omega d\Omega_\gamma} = n_i \int_o^\infty dt \int p^2 dp \int d\Omega_e \frac{p}{\gamma} G(p, \mu_e, \varphi_e, p_o, \mu_o, \varphi_o, t) \hbar\omega \frac{d^2\sigma_{Brems}}{d\omega d\Omega_b},$$

where $\frac{d^2\sigma}{d\omega d\Omega_b}$ is the differential cross section per unit photon frequency and per unit solid angle (θ_b is the relative angle between the electron momentum and the photon momentum), Ω_e is the direction of the electron momentum and Ω_γ is the direction of the emitted photon with respect to initial electron beam direction. Hence the conversion efficiency from relativistic kinetic energy to X ray radiation can be calculated with the help of the Green function G. For example if we consider the total efficiency integrated over the relative angle between the electron momentum and the photon momentum and phton energy we obtain:

$$\frac{E}{\gamma_o - 1} = \frac{\alpha}{\pi} \frac{Z}{\log \Lambda_{ee}} \left[\log \left(183 Z^{-1/2} \right) + \frac{1}{18} \right] \frac{\gamma_o - 1}{\gamma_o}.$$

where $\alpha = \frac{1}{137.04}$ is the fine-struture constant. This type of calculation can be generalized to evaluate the efficiency in a given bandwidth and in a given direction; the final result provides a straightforward method to optimize the conversion with respect to the jet and target parameters.

References:

[1] I. Kostioukov and J.M. Rax, Phys. Rev. Letters, 83, 2206 (1999).
[2] S.T. Beliaev and G. I. Budker, Dokl. Akad. Nauk. SSSR, 107, 807 (1956).
[3] B.J. Braams and C.F.F. Karney, Phys. Rev. Letters, 59, 1817 (1987).
[4] J.M. Rax, J. Robiche and I. Kostioukov, Physics of Plasmas, 6, 3233 (1999).