Runaway Source Term as an Escape Process in a Potential.

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INTRODUCTION

The evolution of the electron distribution function in the momentum space is mainly determined by Coulomb collisional processes with the background of external fields and can be described by the Fokker-Planck equation:

$$\frac{\partial f_e(\boldsymbol{v},t)}{\partial t}+\frac{1}{m_e}q_e\left(\boldsymbol{E}+\frac{\boldsymbol{v}\times\boldsymbol{B}}{c}\right)\frac{\partial}{\partial\boldsymbol{v}}f_e=\frac{\partial}{\partial v_i}\left[-A_i(\boldsymbol{v})\,f_e(\boldsymbol{v},t)+\frac{1}{2}\frac{\partial}{\partial v_k}B_{ik}(\boldsymbol{v})\,f_e(\boldsymbol{v},t)\right]$$

$$A_i(\boldsymbol{v})=\frac{1}{4\pi}\frac{\partial}{\partial v_i}L^{ei}\frac{m_e+M_i}{M_i}H^i(\boldsymbol{v});\quad H^i(\boldsymbol{v})=\int d\boldsymbol{v}_1\frac{f_i(\boldsymbol{v})}{|\boldsymbol{v}-\boldsymbol{v}_1|}$$

$$B_{ik}(\boldsymbol{v})=\frac{1}{4\pi}\frac{\partial^2}{\partial v_i\partial v_k}L^{ei}G^i(\boldsymbol{v});\qquad G^i(\boldsymbol{v})=\int d\boldsymbol{v}_1 f_i(\boldsymbol{v})|\boldsymbol{v}-\boldsymbol{v}_1| \tag{1}$$

$$L^{ei}=\Lambda\left(\frac{4\pi q_e q_i}{m_e}\right)^2$$

With **E** and **B** the electric and magnetic field **v** the velocity, m_e the electron mass, M_i the ion mass, $q_{e/i}$ the electron/ion charge, $f_{e/i}$ the distribution function of electrons/ions and Λ the coulomb logarithm. This description is equivalent to the Langevin approach that allows us to derive the equations for particle motion. The Langevin equations and the relation between their coefficients and the Fokker-Planck coefficients, with the definition of a stochastic integral by Stratonovich, are (2):

$$\frac{dv_i}{dt}=F_i(\boldsymbol{v})+D_{ik}(\boldsymbol{v})\xi_k$$

$$D_{ik}=(B^{1/2})_{ik}=(B^{1/2})_{ki} \tag{2}$$

$$F_i=A_i-\frac{1}{2}D_{jk}\frac{\partial D_{ik}}{\partial v_j}$$

With F_i the drift coefficient, D_{ik} the diffusion coefficient and $\xi(t)$ a random white noise with the following characteristics (3):

$$\langle\xi(t)\rangle=0\qquad\langle\xi_i(t)\xi_k(t+\tau)\rangle=\delta_{ik}\delta(\tau) \tag{3}$$

APPROACH TO ELECTRON-ION SCATTERING

The Fokker-Planck equation (1) for the electron-ion collisions can be simplified when it is applied to runaway electrons. If we neglect energy exchange between electrons and ions (because M_{ion}>>m_e), treat the electron-ion scattering as just a pitch-angle scattering of the electrons against the infinitely massive ions. The latter are characterized by a distribution f_i, proportional to a delta function, and their velocity is considered approximately zero. The Fokker-Panck equation is reduced to:

$$\frac{\partial f_e(\mathbf{v},t)}{\partial t} + \frac{1}{m_e} q_e E_i \frac{\partial}{\partial v_i} f_e = \frac{\partial}{\partial v_i}\left[-a\frac{v_i}{v^3} f_e(\mathbf{v},t) + \frac{1}{2}\frac{\partial}{\partial v_k} a \frac{v^2\delta_{ik} - v_i v_k}{v^3} f_e(\mathbf{v},t)\right]$$
$$a = \frac{4\pi Z_{eff} \Lambda e^4 n_e}{m_e^2} \qquad (4)$$

With n_e the electron concentration, Z_{eff} the ions effective charge ($Z_{eff} = \sum_i n_i Z_i^2 / n_e$).

This equation in the Langevin form [1] is written:

$$\frac{dv_{||}}{dt} = -\frac{eE_{||}}{m_e} - \frac{a}{2v^3} v_{||} + \left(\frac{a}{v^5}\right)^{1/2} v_\perp^2 \xi_{||} - \left(\frac{a}{v^5}\right)^{1/2} v_{||} v_\perp \xi_\perp$$
$$\frac{dv_\perp}{dt} = -\frac{eE_\perp}{m_e} - \frac{a}{2v^3} v_\perp + \left(\frac{a}{v^5}\right)^{1/2} v_{||} v_\perp \xi_{||} - \left(\frac{a}{v^5}\right)^{1/2} v_{||}^2 \xi_\perp \qquad (5)$$

The test ensemble electron trajectories in momentum space are obtained averaging the stochastic term of Langevin equations [2]. These trajectories are showed in the figure 1.

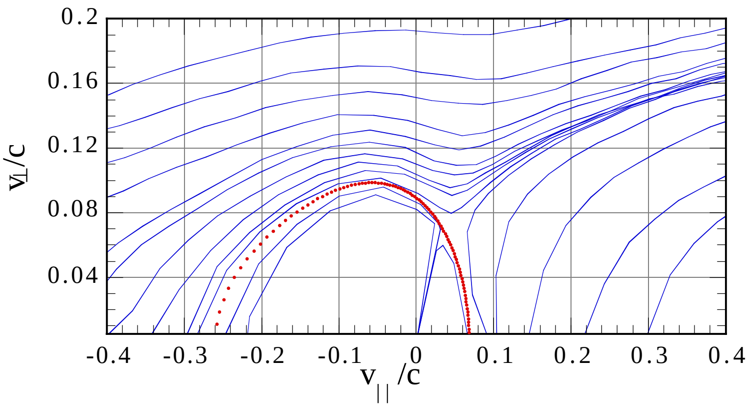

***Figure 1**: Deterministic electron trajectories in momentum space.*

A critical trajectory that separates the runaway and the thermal parts of 2D-momentum space has been obtained from these equations (figure 2). Due to the averaging, the fluctuations are not considered in the equations and then the electrons cannot pass from one zone to another, keeping their character of runaway or bulk electrons.

If we keep the stochastic fluctuations due to collisions in our description, then the electrons can pass from thermal to runaway zone and then generate a source term of runaway electrons. To calculate this flux due to stochastic fluctuations, the electron trajectories also can be considered to obey to a bistable potential field with two wells that correspond to the two stable solutions, thermal and runaway electrons. The second well of the potential is only obtained when synchrotron radiation losses are included, since they provide a mechanism that limits the maximum energy that a runaway electron can reach. The two wells are separated by a barrier, which is situated in momentum space just at the critical trajectory. Then the random force associated to the collisional process can make the electrons jump the barrier and generate a net flux through the critical trajectory in momentum space.

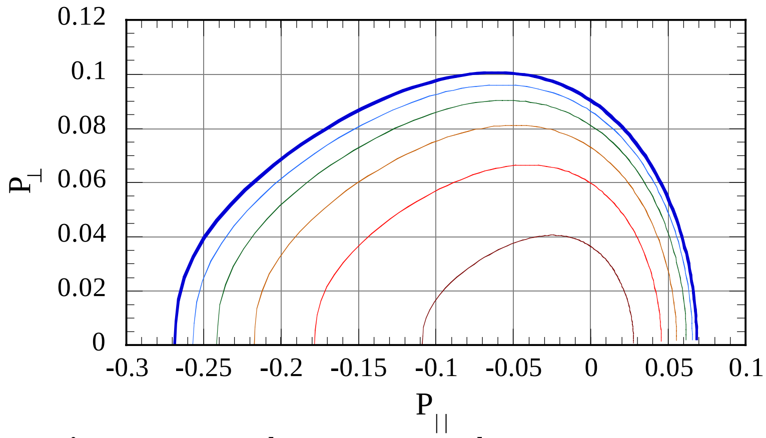

***Figure 2**: Critical trajectories in the momentum space for several values of electric field.*

THE POTENTIAL FORMALISM

We have obtained this potential by simplifying our two dimensional system. We have fixed a local reference system that allows us to express the potential in one dimension. When the B_{ij} matrix is diagonalised to pass from the Fokker-Planck to the Langevin formalism, we obtained that the eigenvalue associated to the eigenvector $\vec{v}$ is zero. Then we can rewrite the Langevin equations in the reference system *(v_1,v_2)*, with $\vec{v} = v_2$ in t=0 and obtain:

$$\frac{dv_1}{dt} = -\frac{eE_1}{m_e c} - \frac{Z_{eff}\Lambda e^4 n_e}{4\pi\varepsilon_0^2 m_e^2 c^3}\frac{v_1}{v^3} + \left(\frac{Z_{eff}\Lambda e^4 n_e}{4\pi\varepsilon_0^2 m_e^2 c^3}\frac{1}{v}\right)^{1/2}\xi_1$$
$$\frac{dv_2}{dt} = -\frac{eE_2}{m_e c} - \frac{Z_{eff}\Lambda e^4 n_e}{4\pi\varepsilon_0^2 m_e^2 c^3}\frac{v_2}{v^3} \qquad (6)$$

Where v is normalized to the speed of light and all the expressions are in SI units.

Near the critical trajectory, $\vec{v}$ is almost parallel to this one, then the fluctuations in the perpendicular direction to the critical trajectory *(v_1)* give us the flux of runaways. This equation have a multiplicative noise force, the diffusion coefficient depends on v_1 through $\vec{v}$. If we want put this equation in potential form we must convert it in an equation with additive noise.

If the following assumptions are fulfilled:

- time independent drift and diffusion coefficients in the Langevin equations
- diffusion coefficient $\neq 0$

the multiplicative noise becomes an additive by a simple variable change [3].

We perform the following change: transform to u_1 with $v_1 = u_1 / v_2^{1/2}$. It can be seen from the expression (6) that a generalized potential Φ related to the u_1 equation, can be defined:

$$\frac{du_1}{dt} = \frac{d}{du_1}\left(-\frac{eE_1}{m_e c} v_2^{1/2} u_1 + \frac{Z_{eff} \Lambda e^4 n_e}{2\pi\varepsilon_0^2 m_e^2 c^3} \left(\frac{u_1}{v_2^{1/2}} + v_2^2 \right)^{-1/4} \right) + \left(\frac{Z_{eff} \Lambda e^4 n_e}{4\pi\varepsilon_0^2 m_e^2 c^3} \right)^{1/2} \xi_1 = \frac{d\Phi}{du_1} + D\,\xi_1 \quad (7)$$

and then the runaway source term can be studied as an escape process [3] in this potential (Fig. 3). If we assume that,

$$\Delta\Phi / D^2 >> 1$$

we can calculate, using the Arrenius relation the escape rate as:

$$r = (2\pi)^{-1} \sqrt{\Phi''(u_{1min}) \left| \Phi''(u_{1max}) \right|}\, e^{-[\Phi(u_{1max}) - \Phi(u_{1min})]/D^2} \quad (8)$$

Where $u_{1min/max}$ are the values where the potential is minimum and maximum, respectively. This expression allows us to study the modification of runaway source due to several factors as current quenching or additional heating.

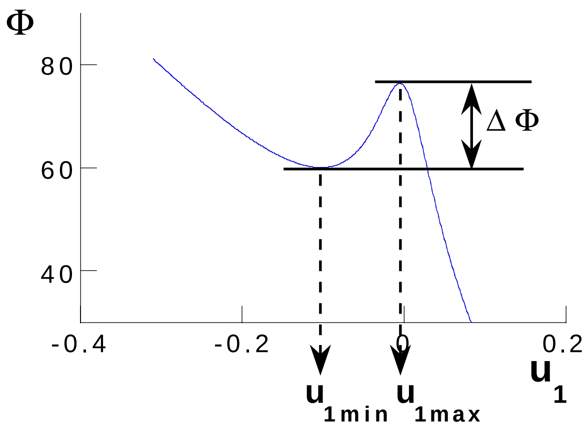

***Figure 3**: Potential associated to u_1*

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