*27th EPS Conference on Contr. Fusion and Plasma Phys. Budapest, 12-16 June 2000 ECA Vol.* **24B** *(2000) 141 -144*

# The Effective Dielectric Tensor for Plasmas with Inhomogeneities in Density and Magnetic Field

Rudi Gaelzer
*Instituto de Física e Matemática, UFPel, Caixa Postal 354 – Campus UFPel, 96010-900 Pelotas - RS, Brazil.*
Luiz F. Ziebell, and Omar J. G. Silveira
*Instituto de Física, UFRGS, Caixa Postal 15051, 91501-970 Porto Alegre - RS, Brazil.*

**Abstract**

We discuss the derivation of explicit expressions for the effective dielectric tensor to be utilized in the dispersion relation for weakly inhomogeneous plasmas. The general expressions obtained are useful for situations with simultaneous existence of weak inhomogeneities in density and magnetic field. The particular case of a Maxwellian distribution in velocity space for the electron population is discussed, and relatively compact expressions for the dielectric tensor are obtained, which display the correct symmetry/antisymmetry properties in the off-diagonal terms and correctly describe the energy exchange between waves and particles. A discussion is made on a future application of this formalism to the problem of cross-field instabilities generated in the magnetotail of Earth's magnetosphere, which can play a significant role in triggering current disruption in the magnetotail, leading to a magnetospheric substorm.

## Introduction

In the present study of the dielectric properties of inhomogeneous plasmas, explicitly incorporating gradients of plasma parameters, the derivation of the components of the dielectric tensor is made along the usual procedure for weakly inhomogeneous media, in which the wave fields are represented by plane-wave approximations. The preliminary dielectric tensor obtained is then corrected, using a procedure introduced by *Beskin, Gurevich and Istomin* (1987) [1] (in what follows referred as BGI transformation). This transformation corrects the plane wave approximation for the dielectric tensor, not valid in an inhomogeneous plasma. As a result, we obtain an *effective* dielectric tensor, which features a series of fundamental properties: 1) The effective dielectric tensor satisfies the Onsager reciprocity relations; 2) the anti-Hermitian part of the effective dielectric tensor, connected to wave absorption, only includes the contribution of resonating particles and doesn't incorporate the physically unacceptable non resonating contributions featured by the tensor obtained from the plane wave approximation, prior the BGI transformation; 3) the effective dielectric tensor is the true Fourier transform of the dielectric tensor in configuration space. These properties are not featured by other formulations found in the literature, which incorporate gradient effects without the BGI transformations. Due to these correct properties, we believe that the effective dielectric tensor is the correct form which must be utilized in the dispersion relation for the study of wave propagation in weakly inhomogeneous plasmas.

We have recently used the formulation of the effective dielectric tensor for the case of magnetic field inhomogeneity, with uniform density and plasma temperature [2–7], while we had previously used the formulation to investigate cases in which the magnetic field is homogeneous and other plasma parameters could be inhomogeneous [8–12]. The main difference between these two situations is that the magnetic field gradient modifies the wave-particle resonance condition, and it is necessary to add corrections to all orders of inhomogeneity to obtain the dielectric tensor with correct properties [6], while for inhomogeneities in other parameters the resonance condition is not affected and a first order correction is sufficient [9].

We now proceed by the development of a unified approach, incorporating both gradients of the magnetic field and of other plasma parameters, which will make possible a considerable increase in the range of phenomena to be investigated. Preliminary results of the more general approach can be found in Gaelzer *et al.*, 1999 [13].

## The effective dielectric tensor for a Maxwellian distribution function

For the derivation of the following expressions, the magnetic field has been considered pointing in the $z$ direction, $\mathbf{B}_0(x) = B_0(1+\epsilon_B x)\hat{\boldsymbol{e}}_z$, with weak inhomogeneities in plasma density and magnetic field along the $x$ direction. The waves were assumed propagating in arbitrary directions, with $k_{||}$ and $k_\perp$ as the components of the wave vector respectively parallel and perpendicular to the magnetic field.

For the distribution function, we will assume a Maxwellian in velocity space, with a linear spatial profile:

$$f_\alpha(u_\perp^2, u_{||}, x) = (1-\epsilon_\alpha x) n_0 g_\alpha(u_\perp^2, u_{||}) = (1-\epsilon_\alpha x) n_0 \frac{\mu_\alpha^{3/2}}{(2\pi)^{3/2}} e^{-\mu_\alpha u^2/2}\,, \tag{1}$$

where

$$g_\alpha(u_\perp^2, u_{||}) = \frac{\mu_\alpha^{3/2}}{(2\pi)^{3/2}} e^{-\mu_\alpha u^2/2}\,, \quad \text{with } \mu_\alpha = \frac{m_\alpha c^2}{T_\alpha}\,.$$

Using this distribution function, one can readily obtain the effective dielectric tensor components from a general expression which includes gradients both in the magnetic field and in particle density. The general expression for the effective dielectric tensor will not be given here due to lack of space. The reader can find it in Ref. [13].

After some calculations, the effective dielectric tensor for the distribution (1) can be given as [13]

$$\overleftrightarrow{\boldsymbol{\varepsilon}} = \overleftrightarrow{\mathbf{1}} + \overleftrightarrow{\boldsymbol{\chi}}_B + \overleftrightarrow{\boldsymbol{\chi}}_P\,, \tag{2}$$

where

$$\begin{aligned}\overleftrightarrow{\boldsymbol{\chi}}_P = {} & i\sum_\alpha \epsilon_\alpha X_\alpha \omega \sum_{n=-\infty}^{+\infty} \int_0^\infty d\tau \int d^3u\, e^{iD_{n\alpha}(0)\tau} u_\perp\, g_\alpha \left[\frac{c}{\omega}\frac{b_\alpha \sin\psi}{\gamma_\alpha} - \mu_\alpha u_\perp x\right] \boldsymbol{\pi}_{n\alpha}^* \boldsymbol{\pi}_{n\alpha} \\ & - i\sum_\alpha \epsilon_\alpha \mu_\alpha X_\alpha \omega \frac{c}{\Omega_\alpha} \sum_{n=-\infty}^{+\infty} \int_0^\infty d\tau \int d^3u\, e^{iD_{n\alpha}(0)\tau} u_\perp^3\, g_\alpha \left[\frac{J_n}{2}\mathbf{e_2}\boldsymbol{\pi}_{n\alpha} + \boldsymbol{\Phi}_{n\alpha}^* \boldsymbol{\pi}_{n\alpha}\right]^H\,,\end{aligned} \tag{3}$$

$$\overleftrightarrow{\boldsymbol{\chi}}_B = i\sum_\alpha X_\alpha \omega \mu_\alpha \sum_{n\to-\infty}^{\infty} \int_0^\infty d\tau \int d^3u\; u_\perp^2 g_\alpha \mathrm{e}^{iD_{n\alpha}(\epsilon_B)\tau} \left[F_{n\alpha}(\tau)\right]^{(|n|-1)} \frac{\boldsymbol{\Pi}_{n\alpha}^- \boldsymbol{\Pi}_{n\alpha}^+}{(W_n^- W_n^+)^{|n|}}\,. \tag{4}$$

The vectors $\boldsymbol{\pi}_{n\alpha}$, $\boldsymbol{\Pi}_{n\alpha}^\pm$ and $\boldsymbol{\Phi}_{n\alpha}$ are given by

$$\boldsymbol{\Pi}_{n\alpha}^\pm = \pm\left[nJ_{|n|}(W_n^\pm) G_{n\alpha}^\pm(\tau) + i\frac{J_{|n|+1}(W_n^\pm)}{W_n^\pm} F_{n\alpha}^{1/2}(\tau) b_\alpha \sin\psi\right]\hat{\boldsymbol{e}}_x$$

$$
\begin{aligned}
&+i\left[|n|J_{|n|}(W_n^{\pm})G_{n\alpha}^{\pm}(\tau) \mp \frac{J_{|n|+1}(W_n^{\pm})}{W_n^{\pm}}F_{n\alpha}^{1/2}(\tau)(b_\alpha\cos\psi \pm \mathcal{K}_n\tau)\right]\hat{\boldsymbol{e}}_y \\
&+\frac{u_\|}{u_\perp}J_{|n|}(W_n^{\pm})F_{n\alpha}^{1/2}(\tau)\hat{\boldsymbol{e}}_z \\
\boldsymbol{\Phi}_{n\alpha} = &\left\{\left[\left(\frac{n^2}{b_\alpha^2}-\frac{1}{2}\right)J_n(b_\alpha) - \frac{J_n'(b_\alpha)}{b_\alpha}\right]\sin(2\psi) - i\left[\frac{n}{b_\alpha^2}J_n(b_\alpha) - \frac{nJ_n'(b_\alpha)}{b_\alpha}\right]\cos(2\psi)\right\}\hat{\boldsymbol{e}}_x \\
&-\left\{\left[\left(\frac{n^2}{b_\alpha^2}-\frac{1}{2}\right)J_n(b_\alpha) - \frac{J_n'(b_\alpha)}{b_\alpha}\right]\cos(2\psi) + i\left[\frac{n}{b_\alpha^2}J_n(b_\alpha) - \frac{nJ_n'(b_\alpha)}{b_\alpha}\right]\sin(2\psi)\right\}\hat{\boldsymbol{e}}_y \\
&+\frac{v_\|}{v_\perp}\left[\frac{n}{b_\alpha}J_n(b_\alpha)\sin\psi + iJ_n'(b_\alpha)\cos\psi\right]\hat{\boldsymbol{e}}_z \\
\boldsymbol{\pi}_{n\alpha} = &\boldsymbol{\Pi}_{n\alpha}^{+}(\epsilon_B = 0) \quad \boldsymbol{\pi}_{n\alpha}^{*} = \boldsymbol{\Pi}_{n\alpha}^{-}(\epsilon_B = 0)\,.
\end{aligned}
$$

The remaining parameters can be found in Ref. [13]. One can easily see that equation (2) obeys Onsager symmetry. This is of fundamental importance, since in this case it is guaranteed that the anti-Hermitian part of the dielectric tensor only contains contributions from the particles that are in resonance with the wave. Such symmetry was not shown by previous works in the literature [13, 16].

## Cross-field instabilities in the Lower Hybrid frequency

Among the various applications of eq. (2), the study of cross-field instabilities in the Lower Hybrid range may be of fundamental importance for the understanding of magnetospheric substorms, since they can be among the most important processes by which the free energy source of the inhomogeneity can be tapped during the substorm growth phase.

We have recently solved the dispersion relation for waves near the Lower Hybrid using the effective dielectric tensor (2). The waves were considered propagating in the $y - z$ plane ($\psi = \pm\pi/2$) and several values of the electron plasma beta were considered, in order to reproduce the physical environment of the magnetotail [17].

Some of the results are shown in Fig. 1a,b. It can be seen that the Modified Two-Stream instability (MTSI) is quickly suppressed by temperature effects, remaining only a region of instability for $q_\| \approx 0$, which could be an indication that in this case remains only the Lower-Hybrid-Drift instability (LHDI), which should be operative for propagation nearly perpendicular to the magnetic field. These results agree with previous work where it has been argued that the LHDI should be the prevailing instability in high electron beta plasmas. For further discussion, the reader is referred to *Silveira et al., 2000* [17].

## Final remarks

We intend to proceed with this unified approach, and use the expressions obtained in the investigation of phenomena which occur in regions where the field gradient is predominantly perpendicular, like the magnetic equator of planetary magnetospheres or the central region of coronal loops in the solar corona and chromosphere. Initial work has been done on the study of cross-field instabilities in a plasma environment similar to the Earth's magnetosphere [17]. Particularly, we are interested in the cross-field drift instabilities that can be important in the growth phase of magnetotail substorms like the MTSI and LHDI [14–17].

**Acknowledgments**

This work has been partially supported by the Brazilian agencies Fundação de Amparo à

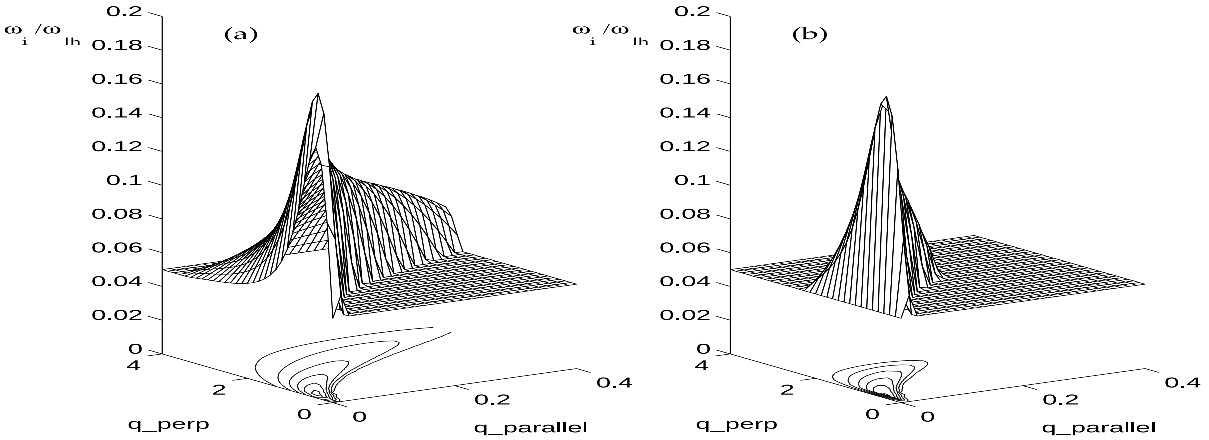


Figure 1: *(a) Growth rate for* $\beta_e = 0.01$. *(b) Growth rate for* $\beta_e = 0.025$. *Here,* $q_\perp = ck_\perp/\omega_e$ *and* $q_{||} = ck_{||}/\omega_e$ *[17].*

Pesquisa do Estado do Rio Grande do Sul (FAPERGS), Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq), and Financiadora de Estudos e Projetos (FINEP).

## References


[1] V. S. Beskin, A. V. Gurevich, and Y. I. Istomin, Sov. Phys. JETP **65**, 715 (1987).

[2] R. Gaelzer, R. S. Schneider, and L. F. Ziebell, in *Segundo Encontro Brasileiro de Física de Plasmas* (PUBLISHER, Serra Negra, SP, Brasil, 1993), pp. 226–229.

[3] R. Gaelzer, R. S. Schneider, and L. F. Ziebell, Proceedings of 1994 International Conference on Plasma Physics **2**, 33 (1994), foz do Iguaçu, Brasil.

[4] R. Gaelzer, R. S. Schneider, and L. F. Ziebell, in *Terceiro Encontro Brasileiro de Física de Plasmas* (PUBLISHER, Águas de Lindóia, SP, Brasil, 1995), pp. 240–243.

[5] R. Gaelzer, PhD thesis, UFRGS, Curso de Pós-Graduação em Física, 1995.

[6] R. Gaelzer, R. S. Schneider, and L. F. Ziebell, Phys. Rev. E **51**, 2407 (1995).

[7] R. Gaelzer, R. S. Schneider, and L. F. Ziebell, Phys. Rev. E **55**, 5859 (1997).

[8] R. A. C. Fº., PhD thesis, UFRGS, Curso de Pós-Graduação em Física, 1990.

[9] R. A. Caldela Filho, R. S. Schneider, and L. F. Ziebell, J. Plasma Phys. **42**, 165 (1989).

[10] R. A. Caldela Filho, R. S. Schneider, and L. F. Ziebell, J. Plasma Phys. **43**, 335 (1990).

[11] C. J. de H. Cavalcanti, MSc Diss., UFRGS, Curso de Pós-Graduação em Física, 1994.

[12] C. J. H. Cavalcanti, R. S. Schneider, and L. F. Ziebell, J. Plasma Phys. **52**, 195 (1994).

[13] R. Gaelzer, L. F. Ziebell, and O. J. G. Silveira. Phys. Plasmas, **6**, 4533 (1999).

[14] C. S. Wu *et al.*, J. Geophys. Res. **97**, 141 (1992).

[15] P. H. Yoon and A. T. Y. Lui, Phys. Fluids B **5**, 836 (1993).

[16] P. H. Yoon, A. T. Y. Lui, and C.-L. Chang, Phys. Plasmas **1**, 3033 (1994).

[17] O. J. G. Silveira, L. F. Ziebell, and R. Gaelzer. Inhomogeneity Driven Instabilities in the Lower Hybrid Range. *Submitted to publication.*