27th EPS Conference on Contr. Fusion and Plasma Phys. Budapest, 12-16 June 2000 ECA Vol. **24B** (2000) 137 -140

LONG TERM EVOLUTION OF MAGNETIC RECONNECTION PROCESSES

D. Grasso[1], F. Califano[2], F. Pegoraro[2], F. Porcelli[1]
[1] *Istituto Nazionale Fisica della Materia and Politecnico di Torino, Italy*
[2] *Istituto Nazionale Fisica della Materia, Università di Pisa, Italy*

Abstract

The nonlinear evolution of Hamiltonian magnetic field line reconnection in a two-dimensional fluid plasma leads to an equilibrium with a macroscopic island and fine-scale spatial structures. The latter arise from the phase mixing of the conserved fields. This equilibrium is the analog of the BGK solution for electrostatic Langmuir waves.

Magnetic field line reconnection is one of the most fertile problems in plasma physics. One of its most important features is the interplay between the energetic and the topological aspects that characterise its evolution: a relaxation of the global magnetic structure is accompanied by a local, fast release of magnetic energy. This energy is transformed into heat and into "ordered" electron and ion kinetic energy. The topological features of magnetic reconnection are most evident in the dissipationless (Hamiltonian) regime. In this regime the topology of the magnetic field is broken by the effect of electron inertia, but the topology of generalised fields is preserved as discussed in Refs. [1]. While the existence of these conserved fields has been shown not to limit the evolution of magnetic reconnection, and in particular the value of the reconnected magnetic flux, their effect on the asymptotic state of the magnetic configuration and on the eventual formation of a "macroscopic" equilibrium has not been fully investigated.

Magnetic reconnection processes due to instabilities were studied in the collisionless limit in Refs. [1]-[2] in 2D configurations with double periodic boundary conditions. An intrinsic limitation of that approach was the "cross-talking" between island chains with a width comparable to the equilibrium scale length. In this paper we remove the double periodic boundary conditions. We adopt a Harris-type equilibrium configuration with a strong superimposed magnetic field in the third direction. The integration domain extends to infinity in the direction of the equilibrium inhomogeneity. With these boundary conditions, a single coherent magnetic island can be followed in time, until its width saturates at a macroscopic value. The saturation mechanism is associated with a reversible energy transport to small scale structures in the fluid vorticity and in the current density inside the island. These structures are superimposed on the macroscopic equilibrium and arise as a consequence of the phase mixing inside the island of the conserved fields. In this way a macroscopic equilibrium can be accessed by a Hamiltonian plasma in spite of energy conservation.

We consider a 2D configuration with a strong magnetic field in the ignorable coordinate z, $\mathbf{B} = B_0\mathbf{e}_z + \nabla\psi \times \mathbf{e}_z$ where B_0 is constant and $\psi(x, y, t)$ is the magnetic flux function. The governing equations, normalised on the Alfvén time τ_A and on the equilibrium scale length, can be cast in the Lagrangian invariant form [6]

$$\frac{\partial G_\pm}{\partial t} + [\phi_\pm, G_\pm] = 0, \tag{1}$$

with Hamiltonian $H = -\int d^2x(\phi_+G_+ + \phi_-G_-)/2$. Here $[A, B] = \mathbf{e}_z \cdot \nabla A \times \nabla B$, and the conserved fields are the Lagrangian invariants

$$G_\pm \equiv \psi - d_e^2\nabla^2\psi \pm d_e\varrho_s\nabla^2\varphi \tag{2}$$

which are advected along the stream lines of

$$\phi_\pm \equiv \varphi \pm (\varrho_s/d_e)\psi. \tag{3}$$

The magnetic flux ψ and the plasma stream function φ, which define the velocity field $\vec{v} = \vec{e}_z \times \vec{\nabla}\varphi$, obey the equations $\psi - d_e^2\nabla^2\psi = (G_+ + G_-)/2$ and $d_e\varrho_s\nabla^2\varphi = (G_+ - G_-)/2$, with d_e the electron collisionless skin depth and ϱ_s the so called ion sound gyro-radius[1]. The connection with the more standard form of the governing equations, as derived from the two fluid plasma description in Ref. [7] and used in Refs. [1], is given e.g., in Ref.[6]. In particular, $J = -\nabla^2\psi$ is the current density and $U = \nabla^2\varphi$ the plasma fluid vorticity along z. In the cold electron limit $\varrho_s \to 0$, the Lagrangian invariants become degenerate and only the generalised magnetic flux $F \equiv \psi - d_e^2\nabla^2\psi$ admits a Lagrangian conservative equation with stream function φ. The equilibrium magnetic field is $\mathbf{B}_{eq} = B_{z0}\mathbf{e}_z + \mathbf{B}_{yeq}(x)\mathbf{e}_y$, where $\mathbf{B}_{yeq}(x) = \tanh(x/L)$ and L is the equilibrium scale length. This equilibrium is unstable to tearing perturbations, which are periodic in y over the distance L_y, when $L \leq \pi L_y$. In the x direction we impose the perturbation fields $\delta\psi$ and $\delta\varphi$ to vanish at the edge of the integration domain. The model preserves parity, so we choose initial perturbations such that $\psi(-x, y) = \psi(x, y)$ and $\phi(-x, y) = -\phi(x, y)$. These relations imply $G_+(-x, y) = G_-(x, y)$, $\phi_+(-x, y) = -\phi_-(x, y)$ and $H = 0$.

Eqs. (1) have the form of two coupled Vlasov equations (with x and y playing the role of the coordinate and of the conjugate momentum) for the "distribution functions" $G_\pm$ of two "particle" species with opposite charges in the Poisson-like equation for ϕ and equal charges in the Yukawa-like equation for ψ. Somewhat similarly to Vlasov equation, the relationship between the distribution functions $G_\pm$ and the fields $\nabla\phi_\pm$ appearing in the "acceleration " terms in Eqs. (1) involves integration over the velocity (momentum) variable. This analogy turns out useful for the interpretation of the system evolution. Eqs. (1) are integrated numerically for the following set of parameters: $L/L_y = 1/4\pi$, $d_e = \varrho_s = 0.2$ for Hamiltonian reconnection and $\eta = 0.002$, $\varrho_s = 0.1$ for resistive reconnection.

We see in Figs. 1 and 2 that after a super exponential phase, already found in Refs. [1]-[2], the growth of the island saturates and magnetic energy is transformed mainly into plasma kinetic energy. Here, we define the total energy $\mathcal{E}$ as

$$\mathcal{E} \equiv \int d^2x \left(|\nabla\psi|^2 + d_e^2J^2 + \varrho_s^2U^2 + |\nabla\varphi|^2\right)/2. \tag{4}$$

where the difference between $\mathcal{E}$ and the Hamiltonian H is a (conserved) quadratic functional of the Lagrangian invariants $G_\pm$ (see Refs. [6], [1]).

In the equilibrium configuration the $G_\pm$ isocontours are simply $y = const$ lines. As the instability evolves, these lines are transported along the characteristic curves, $\vec{x}_\pm(t)$, determined by $d\vec{x}_\pm(t)/dt = \vec{v}_\pm(\vec{x}_\pm, t)$, with $\vec{v}_\pm = \vec{e}_z \times \nabla\phi_\pm$. Therefore, two vortical patterns (one the mirror image of the other) develop inside the magnetic island. These patterns lead to the phase-mixing of $G_\pm$, as shown in Fig. 2. This phase mixing causes the formation of the typical "quadrupolar" fine spatial structure with odd symmetry for the plasma vorticity $U = (G_+ - G_-)/(2\rho_s d_e)$ and, with even symmetry, for the generalized flux function $F = (G_+ + G_-)/2$. A new characteristic time related to the "eddy" turn-over time inside the island becomes of interest. This time scale is shorter as the instability grows and can be estimated to be inversely proportional to the square root of the amplitude of the perturbed fields. When the turning time becomes shorter than the nonlinear island growth time, energy is removed effectively from the large spatial scales leading to the island growth saturation. In order to show that this process can make it possible access to a new "macroscopic" stationary state we proceed along lines that are similar to those followed in Ref. [3] for the Bernstein-Greene-Kruskal (BGK) solutions in the case of the nonlinear Landau damping of Langmuir waves. We separate the Lagrangian invariants into a coarse grained and a phase-mixed part according to $G_\pm = \bar{G}_\pm + \tilde{G}_\pm$. Then by solving the Poisson and the Yukawa equations that relate $G_\pm$ to φ and ψ, we see that when the turning time is shorter than the growth time, $\varphi \approx \bar{\varphi}$ and $\psi \approx \bar{\psi}$, since the contributions of the phase-mixed parts $\tilde{G}_\pm$ cancel out. On the contrary these parts continue to contribute to the total energy conservation through the $d_e^2 J^2$ and $\varrho_s^2 U^2$ terms. This makes it possible for the coarse grained quantities to reach a macroscopic equilibrium given by $[\bar{\phi}_\pm, \bar{G}_\pm] = 0$ without violating energy conservation. Together with the symmetry conditions , this equation gives

$$\bar{\psi} - d_e^2 \nabla^2 \bar{\psi} \pm d_e \varrho_s \nabla^2 \bar{\varphi} = \mathcal{G}(\bar{\psi} \pm (d_e/\varrho_s)\bar{\varphi}). \tag{5}$$

We assume, consistently with the numerical results, that $\bar{\psi}$ is the dominant term on both sides of Eq. (5), as indeed is the case for large x. Then, the arbitrary function $\mathcal{G}$ must be of the form $\mathcal{G}(A) \approx A + d_e^2 \mathcal{F}(A)$, where d_e^2 plays the role of the smallness parameter. Thus, by expanding Eq.(5), we find to leading order that $\bar{\varphi}$ vanishes and $\bar{J} = \mathcal{F}(\bar{\psi}$ (we assume $\varrho_s/d_e \approx O(1)$). The functional dependence of $\bar{J}$ on $\bar{\psi}$ is not determined by Eq. (5) and should be obtained from the nonlinear evolution of the instability. It is remarkable that the function $\mathcal{F}$ need not contain either d_e or ϱ_s and may indeed be determined by geometry. This may explain why the saturated island width is found to be the same as the one we obtain with a resistive Ohm's law.

In conclusion, the analysis of the long term evolution of reconnection processes has revealed the underlaying unity between different physical phenomena such as nonlinear Landau damping of Langmuir waves[3, 4] and dissipationless vortex dynamics[5] in 2D fluids on one side, and Hamiltonian magnetic field line reconnection on the other.

References

[1] E. Cafaro, D. Grasso, F. Pegoraro, F. Porcelli, A. Saluzzi, *Phys. Rev. Lett.* **80**, 4430 (1998).

[2] M. Ottaviani and F. Porcelli, *Phys.. Rev. Lett.* **71**, 3802 (1993).

[3] C. Lancellotti and J.J. Dorning, *Phys.. Rev. Lett.* **81**, 5137 (1998).

[4] M. Brunetti, F .Califano, F. Pegoraro to appear in Phys Rev E (2000).

[5] J. Miller, P.B. Weichman, and M.C. Cross, Phys. Rev. A **45** 2328 (1992)

[6] B. N. Kuvshinov, V. Lakhin, F. Pegoraro and T. J. Schep, Journ. Plasma Phys. **59** (4) (1998).

[7] B.N. Kuvshinov, F Pegoraro and T. Schep, *Physics Letters A* **191**, 296 (1994).

(a) (b)

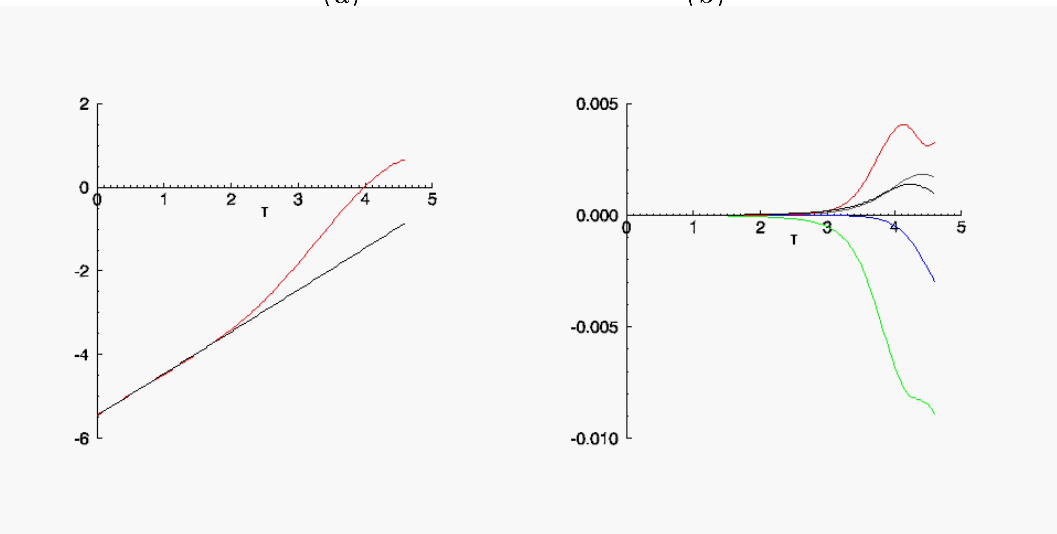

Figure 1: Fig. (a): $\ln(\delta\psi_X)$ versus the normalized time, $T = t * \gamma_L$ with $\gamma_L = 0.02827$. The black line corresponds to the linear regime growth rate extrapolated to nonlinear times. Fig. (b): the different term of the energy as defined in 4, normalized to the total $\mathcal{E}(0)$, are plotted versus the normalized time, T . The blue line is the total energy; the green line is the magnetic energy; the black line is the electron kinetic energy; the gray line is the electron internal energy; the red line is the plasma kinetic energy

.

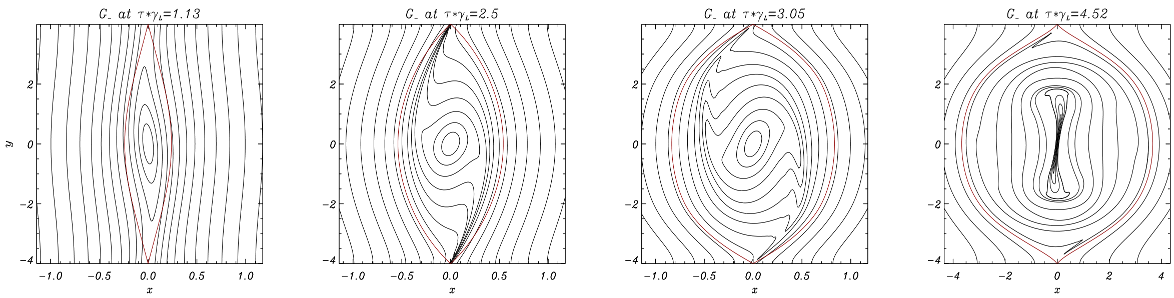

Figure 2: Contour plots of G_- at different simulation times. The separatrix of the magnetic flux, ψ, is in red

.