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Vortex Motion in a Pure Electron Plasma

M. Romé[1], M. Amoretti[1], M. Brunetti[2], F. Califano[3], F. Pegoraro[2] and R. Pozzoli[1]

[1]INFM (U.d.R. Milano Università) and Dipartimento di Fisica, Università degli Studi di Milano, Via Celoria 16, 20133 Milano, Italy

[2]INFM (U.d.R. Pisa) and Dipartimento di Fisica, Università di Pisa, P.za Torricelli 2, 56126 Pisa, Italy

[3] INFM (U.d.R. Pisa), P.za Torricelli 2, 56126 Pisa, Italy

The electrons confined in a Penning-Malmberg trap [1] rotate in the plane perpendicular to the applied static and uniform magnetic field due to the radial electric field produced by their uncompensated electric charge, and bounce back and forth along the magnetic field lines, being confined at the two ends of the trap by the applied electrostatic field. In a large range of operational regimes, the bounce averaged motion of the electrons is much faster than their rotation in the perpendicular plane. In such a case the bounce averaged motion becomes essentially independent of the axial bounce length. In this limit the plasma dynamics becomes isomorphic to that of a two-dimensional (2D) incompressible inviscid fluid, where the fluid vorticity corresponds to the electron density and the stream function to the electrostatic potential [2]. The 2D drift-kinetic equations for a uniform magnetic field at zeroth order in the guiding center approximation are written, using polar coordinates (r, θ), as

$$\left(\frac{\partial}{\partial t} - \frac{1}{r}\frac{\partial\Phi}{\partial\theta}\frac{\partial}{\partial r} + \frac{1}{r}\frac{\partial\Phi}{\partial r}\frac{\partial}{\partial\theta}\right) n(r,\theta,t) = 0; \tag{1}$$

$$\left(\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\right)\Phi(r,\theta,t) = n(r,\theta,t). \tag{2}$$

Here, r is normalized over the radius R of the outer circular container, the density, n, over a characteristic density, $\bar{n}$, the potential, Φ, over $4\pi e\bar{n}R^2$ and the time, t, over ω_c/ω_p^2, with ω_p the electron plasma frequency computed with the characteristic density $\bar{n}$. The boundary condition for the potential is given at the wall of the outer circular conductor (this corresponds to a free-slip boundary condition in the fluid analog).

Various aspects of the 2D dynamics of extended vortices in a pure electron plasma have been studied by means of different numerical approaches. Here, two specific situations are investigated: i) the motion of a single vortex or of several vortices in an inhomogeneous, centrally peaked, background plasma density, by means of a recently developed 2D electrostatic fluid code [3], which solves directly Eqs. (1)–(2); ii) the interaction between an extended vortex with one or several very localized vortices, by means of the particle in cell (PIC) code XOOPIC [4].

Eqs. (1)–(2) are Hamiltonian and express the fact that the electron density is a Lagrangian invariant, i.e., it is constant along the (nonlinear) characteristics given by the

$\mathbf{E} \times \mathbf{B}$-drift. The evolution of n described by Eqs. (1)–(2) is constrained by an infinite number of constants of motion, which correspond to the conservation of the spatial integral of any arbitrary (smooth) function of the Lagrangian invariant density. These topological invariants are conserved in addition to the dynamical constants of motion, i.e., to the energy and the canonical angular momentum of the plasma. In the absence of dissipation, such Hamiltonian equations with an infinite number of constraints are known to develop increasingly smaller scales during their nonlinear evolution. In the case of pure electron plasmas this corresponds to the formation of increasingly thinner density filaments. PIC integration of the electron density evolution resolves in general this small scale formation automatically by introducing an intrinsic effective "coarse graining". This causes a non-conservation of the topological invariants (with the obvious exception of the linear invariant $\int r dr d\theta n$, which corresponds to the conservation of the total number of particles). The developed fluid code, on the contrary, conserves the topological invariants which constrain the 2D plasma dynamics quite accurately. Very small localized violations of the conservations may still arise due to finite spatial resolution and/or the effect of the filtering [5] introduced in the numerical scheme to assure stability. The code has been applied in particular to the study of the motion of a single vortex or of several vortices in a centrally peaked, background plasma density. Vortices corresponding to local density enhancements (clumps) tend to propagate inwards, while vortices corresponding to local density depressions (holes) tend to propagate outwards as a consequence of the conservation of the (canonical) angular momentum of the system. The clump inward propagation against the background vorticity gradient is accompanied by an inward motion of lower density regions. Small scales are formed and localized density (i.e., vorticity in the fluid analog) reconnection gives rise to dipole-type configurations. The occurrence of these reconnection events is registered by small variations of the ideal topological invariants [3]. A time sequence of the evolution of three clumps is shown in Fig. 1. In the initial phase, regions of lower density are carried inward together with the clumps and grooves are cut in the background density distribution. Density reconnection occurs when the sides of the grooves are brought close together by stretching, leading to separated positive and negative vortices (the vortex sign being defined with respect to the local average density value). At later times the low density regions evolve into separate holes which move outwards descending the background vorticity gradient, while the high density regions still moving towards the center are brought close together and tend to coalesce.

Vortex merger and the interaction of a vortex with a vorticity background are basic processes in 2D decaying turbulence. The merging process, in particular, can be characterized theoretically by studying the relatively simple situation of the interaction between an extended vortex with one very localized ("point") vortex. This scenario has been investigated numerically by means of the (2D in real space, 3D in velocity space) PIC code XOOPIC [4]. A comparison has been made with the theoretical model of Ref. [6], based on a suitable Hamiltonian formulation of the dynamics of the point vortex. In the model of Ref. [6], two uniform density, circular vortices with vorticities n_{point} and n_{ext}, and radii r_{point} and r_{ext}, respectively, are considered. The point vortex orbits around the extended vortex, whose displacement is assumed to be negligible. The model is characterized by the condition $1 \ll n_{\mathrm{point}} \ll (r_{\mathrm{ext}}/r_{\mathrm{point}})^2$. Here and in the following, the density of the extended vortex, n_{ext}, is assumed as characteristic density, $\bar{n}$. While orbiting, the point vortex excites waves on the surface of the extended vortex, and these waves, in turn, influence the dynamics of the point vortex. The most efficient excitation of a surface wave takes place when the orbital frequency of the point vortex around the extended vortex, $\omega_{\rm rot} = (1/2)(r_{\rm ext}/r)^2 + n_{\rm point} r_{\rm point}^2/[2(1-r^2)]$, and the frequency of the surface wave, $\omega_m = (m-1+r_{\rm ext}^{2m})/2$, satisfy the resonance condition, $\omega_m - m\omega_{rot}(r) = 0$, where m is the azimuthal wave number, and (r, θ) are the polar coordinates of the point vortex position. With respect to Ref. [6], here the effect of the boundary condition on the circular container, $\Phi(r=1) = 0$, has been taken into account. Critical layers around the extended vortex can be introduced where "point vortex-surface wave" resonances occur. Vortex merger occurs only when there is an overlap of neighboring critical layers. In this case the point vortex cascades through a sequence of resonances as it spirals in and merges with the extended vortex. This scenario of cascading excitation of the resonant surface waves on the extended vortex is confirmed by the numerical simulations. A time sequence of the evolution of a "point vortex" and of an extended vortex in interaction with each other is shown in Fig. 2. To obtain these results, the XOOPIC code has been used in the electrostatic approximation, and in Cartesian geometry. The boundary condition for Φ has been assigned on a polygonal of mesh segments suitably simulating a circumference of radius R. Both vortices have been given a low temperature Maxwellian distribution, and the parameters of the simulation have been chosen in order to fit the requirements of the strongly magnetized plasma approximation. The point vortex tends initially to spiral in the critical layer around the radius corresponding to the excitation of a certain wave, and only when the higher harmonics surface waves are sufficiently strong, the approach to merger begins. Interestingly, it is found that if the intensity of the point vortex is sufficiently high, the point vortex merges in the extended vortex without being sheared apart (i.e., the point vortex keeps its "identity": an average radius of the point vortex can still be defined, and no particles are lost). The extended vortex strongly modifies its shape to capture the point vortex, and when merger occurs, a strong differential shear develops, which produces long filaments in its external region. As time goes on, an "halo" of particles tends to be ejected from the extended vortex, while a new core region can be identified, which in its interior contains the point vortex (surrounded and separated from the extended vortex by an empty annular region). These scenarios are qualitatively confirmed by recent experimental results obtained in the "Vortex" Penning-Malmberg trap at the University of California, in Berkeley [7].

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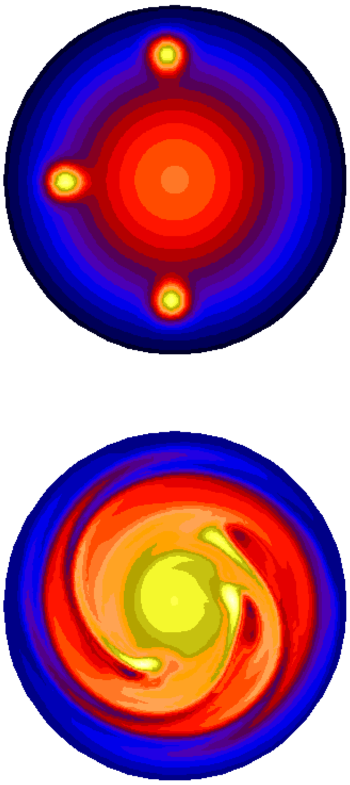

Figure 1: Time evolution of 3 clumps in an inhomogeneous density background, $n(r) = \exp(-r^2)$, obtained from 2D fluid simulations. From left to right: t=0, 22, 48, 102.

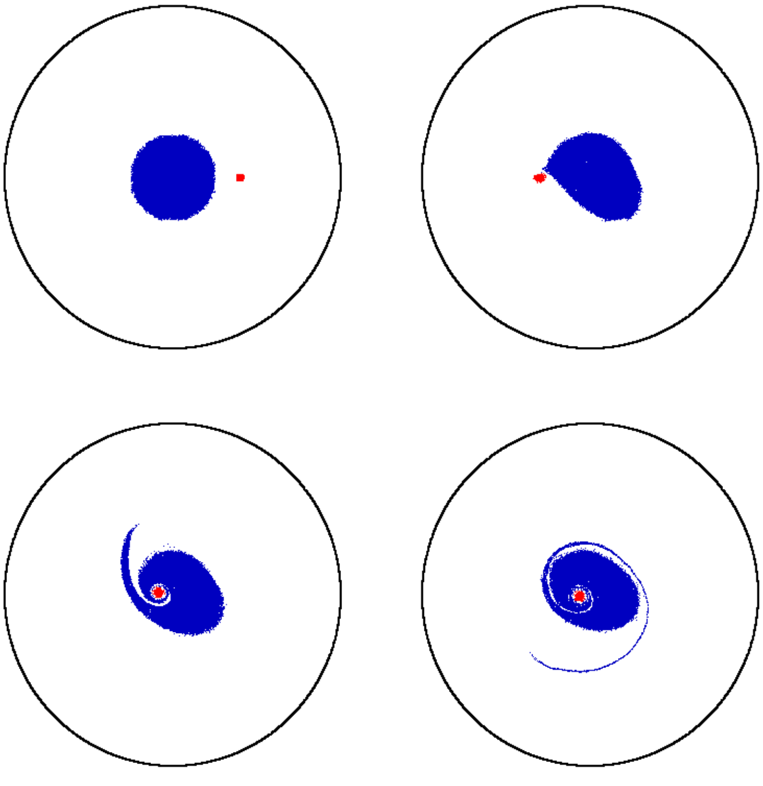

Figure 2: Time evolution of the interaction of a point vortex with an extended vortex, obtained from PIC simulations. From left to right: t=0, 14, 28, 42.