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Non–gaussian probability distribution functions in two dimensional Magnetohydrodynamic turbulence

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Abstract

We analyze intermittency in MHD turbulence using high resolution two–dimensional numerical simulations. We find that the shape of the Probability Distribution Functions (PDFs) of the fluctuations of the magnetic and velocity field strongly depend on the observed scale. A model by Castaing *et al.* [1] has been used to fit the experimental PDFs. We then characterize the intermittency in the MHD fields by the scaling properties of the parameter λ_r^2 obtained from the fits. This method lead us to compare the intermittency properties in different MHD turbulent flows.

Intermittency is one of the most important features of plasma turbulence [2, 3]. The equations of the magnetohydrodynamic, as well as in the case of fluids, display scale invariance, which lead to a self–affine behavior of the fields. The most impressive effect of such behavior is the non–gaussian statistics of the fluctuations of the fields, which is scale dependent. In the framework of the multiplicative cascade, the presence of intense fluctuations of the fields becomes more probable at small scales, because of the progressive localization of the active structures. The tails of the PDFs become higher than gaussian as the scale decreases, and rare events become significant in the statistics. Recent works pointed out that such effects are present in several situations, as for example in the solar wind plasma [4] and in laboratory plasma [5]. In this paper we analyze high resolution two–dimensional numerical simulations, in order to compare the intermittency statistical properties with those observed in "real" plasmas. To do that we use the PDFs of the fields fluctuations $\delta\psi_r$, where $\boldsymbol{\psi}'$ can be either the velocity field $\boldsymbol{v}$ or the normalized magnetic induction $\boldsymbol{b} = \boldsymbol{B}/\sqrt{4\pi\rho}$ (ρ is the constant density); $r = |\boldsymbol{r}|$ is the scale under analysis.

The data we use are magnetic and velocity fields from two–dimensional MHD incompressible simulations (see [6, 7] for details), with periodic boundary conditions at a resolution of 1024^2 grid points. The data sets consist of about 10^7 points, and up to now, these are the largest data sets used to investigate intermittency in statistically steady MHD flows.

We computed the experimental PDFs for different values of the scale $10^{-3} \leq r/L \leq 0.5$, where $L = 2\pi$ is the length of the simulation box, in order to study their scaling properties. In figure we show the PDFs of the normalized fluctuations of the magnetic field; the same phenomenology holds for the velocity field. Such an intermittent behavior, that is the departure from the large scale gaussian PDF, in the framework of the Kolmogorov's refined similarity hypothesis [2], can be attributed to the fluctuations of the energy transfer rate. Following this idea, we observed that the dependence on scales of the PDFs is eliminated when looking at the PDFs conditioned to a given value of the energy transfer rate at the scale r [8, 9]. For each scale r, the intermittency can be described as a superposition of different PDFs of fluctuations each one belonging to subsets with different transfer rates. In the multifractal framework [2, 10, 11], we give a quantitative analysis of the continuous scaling of PDFs. Using a model by Castaing *et al.* (see [1, 4, 5, 7] for details), we can extract directly from the experimental observations a small set of parameters useful to characterize the intermittency. In this model, the PDFs of fluctuations at scale r can be described as a convolution of the large scale (gaussian) distribution with a distribution for $\varepsilon_r^{\pm}$. In the model [1], this can be done by introducing the distribution $\mathcal{L}_{\lambda_r}(\sigma_r)$ for the standard deviation σ_r of the gaussians:

$$P(\delta\psi_r) = \int_0^\infty \mathcal{L}_{\lambda_r}(\sigma_r) \exp\left(-\delta\psi_r^2/2\sigma_r^2\right) \frac{1}{\sqrt{2\pi}\sigma_r} \frac{d\sigma_r}{\sigma_r} \, . \tag{1}$$

where we use a log–normal distribution, as in Castaing et al. [1]:

$$\mathcal{L}_{\lambda_r}(\sigma_r) = \frac{1}{\sqrt{2\pi}\lambda_r} \exp\left(-\frac{\ln^2 \sigma_r/\sigma_{0,r}}{2\lambda_r^2}\right) \, . \tag{2}$$

The parameter $\sigma_{0,r}$ is the most probable value of the σ_r deviations for a given scale r, while the parameter λ_r represents the width of the log–normal distribution $\mathcal{L}_{\lambda_r}(\sigma_r)$. The parameter λ_r is the one characterizing the shape of the PDF, and we can use it for a quantitative measurement of the intermittency.

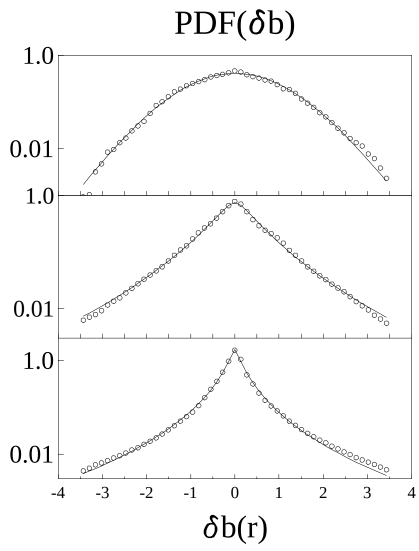

Figure 1. *The PDFs of the fluctuations of the normalized magnetic field* $\boldsymbol{b}$ *for different scales (from the top:* $r/L = 0.002$*,* $r/L = 0.03$ *and* $r/L = 0.25$*). The full line is the fit with the convolution (1).*

The results of the fit of the PDFs with the model (1) are shown in figure . We now look at the scaling of the parameter λ_r^2 with the separation length r/L (figure) in a range that can be seen as the inertial range, between the dissipative saturation and the gaussian regime. For the velocity field, we find a smaller scaling range (see table) than for the magnetic field. We then figured out the scaling parameters from the power–law scaling:

$$\lambda_r^2(r) \sim (r/L)^{-\beta} \,, \tag{3}$$

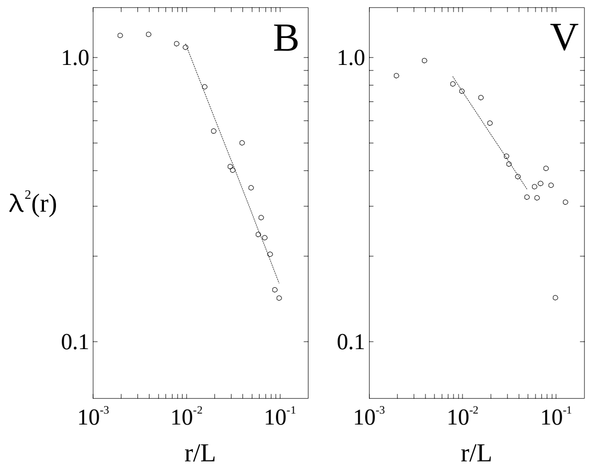

Figure 2. *Scaling behavior of the exponent λ_r^2 for the magnetic and velocity fields (see insert). Straight lines represent the fit with power-laws.*

The scaling exponent β and the maximum value of λ_r^2 over r, λ_{max}^2, are now able to describe quantitatively the intermittency. In fact, the parameter β represents how "fast" the intermittency develops and the wings of the distribution of $\delta\psi_r$ increase. The parameter λ_{max}^2 tells us how "strong" is the intermittency, *i.e.* how deeply the intermittent cascade is active and generates the strongest events $\delta\psi_r$. We report in table the results of the fit of data with the power–law 3. The magnetic field is more intermittent than the velocity, in agreement with previous numerical results [6, 12], and with the results in solar wind plasmas [4]. In MHD the stronger intermittency of the magnetic field is perhaps due to the fact that the magnetic field behaves like a passive vector.

We are now able to compare our results with some previous experimental analysis. The values of β obtained here are larger than those found for fluid flows ($\beta \simeq 0.3$) [1], for solar wind turbulence ($\beta \simeq 0.2$) [4] and for magnetic turbulence in a Reverse Field Pinch laboratory plasma ($\beta \simeq 0.4$) [5]. Thus, 2D MHD turbulence in our numerical simulations is stronger than in geophysical or laboratory plasmas. The values of λ_{max}^2 found here are also greater than those found in the solar wind (slightly so for the magnetic field, more clearly for the velocity) [7], and the magnetic field is strongly more intermittent than in the laboratory plasma mentioned above.

In summary, this paper shows that the analysis of the PDFs can be used to quantitatively characterize the intermittency, through only two parameters, β and λ_{max}^2. The results we found are consistent with previous analysis in 2D MHD simulations, and a comparison with real plasmas has been done.

Table I. *For the magnetic and velocity fields, values of the two parameters* β *and* λ^2_{max}*, together with their statistical error bars, determined from the indicated ranges of fit (see text).*

	β	λ^2_{max}	range of fit
v	0.50 ± 0.15	0.8 ± 0.2	$0.008 \leq r/L \leq 0.05$
b	0.84 ± 0.13	1.1 ± 0.3	$0.01 \leq r/L \leq 0.1$

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