Radial Electric Field Generation During Anomalous E x B Ion Diffusion

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Introduction

In our foregoing paper [1], an anomalous $\mathbf{E} \times \mathbf{B}$ impurity diffusion in a stationary, spatially periodical potential field has been described. In [2], an idea of M. Tendler to use this phenomenon for creating radial electric fields on rational surfaces of tokamaks, where magnetic islands can appear, has been presented. It was assumed that potential perturbations generated within islands should form periodical potential landscapes similar to our model [1]. According to Tendler, the diffusion of impurities in the tokamak edge plasma can have a similar effect. We discuss this in test-particle and PIC simulations.

Potential structures, generated by plasma turbulence, are observed in tokamaks (e.g. [3,4]), with poloidal correlation lengths $\lambda \approx 10 - 20\ mm$, lifetimes τ several tens of microseconds (in [4], $\tau \approx 20 - 50\ \mu s$ was detected), and amplitudes $U < 100\ V$.

Bellan [5] proposed a diffusion mechanism based on the $\mathbf{E} \times \mathbf{B}$ drift motion of ions around blobs (i.e. hills and valleys of the spatially nonhomogeneous electrostatic potential) which grow and decay randomly in time. Recently [1], we suggested an alternative mechanism, which works even when the potential blobs can be considered as *quasistationary.* This mechanism is based on the intrinsic stochasticity regime of particle motion.

Assuming the spatially periodical potential in the cartesian coordinate system x, y to be of the form $U = U_0[2 + \cos kx + \cos ky]$, and the magnetic field $\mathbf{B} = (0, 0, B_z)$ (Fig. 1 presents the system of equipotential lines and separatrices), we have found that in the dimensionless variables $\xi = kx$ and $\eta = ky$ the motion is described, besides its initial conditions, by the parameter $R = Am_pU_0k^2(Zq_pB_0^2)^{-1}$, where A and Z are the impurity mass and charge numbers, and m_p and q_p are the proton mass and charge, respectively. We have found that there can appear regular and diffusive motion, depending on R. Figure 2 ($R = 1.2$) exhibits all possible types of motion: regular motion, random walk type motion, and Lévy walk diffusion. This latter form of diffusion is especially strong for impurities. For $U_0 = 10V, k = 10^{-3}\mathrm{m}^{-1}$ and $B = 1\,\mathrm{T}$, the diffusion coefficient for singly ionized carbon is of the order of $10^1 m^2 s^{-1}$.

Stochasticity of ion motion in the magnetic island region

The possibility to use our model for describing ion diffusion out of magnetic islands is connected with the existence of electrostatic potential perturbations within the magnetic islands. According to [6], the order of this perturbation can be expressed as $q\Phi/kT \approx w/r$, where w is the island width and r is the minor radius of the rational surface. Let us consider, e.g., three rows of islands with $m/n = 6/5$, $5/4$ and $4/3$ and $q = 1 + r^2/a^2$, where a is the minor tokamak radius. Let $a = 2m$. Then rational surfaces with $q = 6/5, 5/4, 4/3$ appear at $r = 0.894\,\mathrm{m}, 1\,\mathrm{m}, 1.15\,\mathrm{m}$, respectively. It is natural to assume that the poloidal periodicity of this potential is the same as of that of the magnetic islands. When the neighbouring rows of islands overlap, the width of the islands is of the order of $0.1\,\mathrm{m}$. For $kT \approx 1\,\mathrm{keV} - 10\,\mathrm{keV}$, the potential Φ can be of the order $\Phi \approx 0.1 - 1kV$.

Generally, the full 3D Hamiltonian must be used. Nevertheless, since it is possible to assume that the gradients of the potential will be significantly larger in the poloidal plane than in the toroidal direction, it is possible, in the first approximation, to consider only the 2D Hamiltonian for the poloidal plane, $H = H(r, p_r, \theta, p_\theta)$, where p_r, p_θ are the canonically conjugated momenta to the coordinates r, θ. First numerical results of our 3D code confirm this possibility.

For a numerical estimate, we choose $\Phi_1(r, \theta) = U_0 \times \cos(k_r r) \times \cos(m\theta)$, where $\lambda_r = 2\pi/k_r$ simulates the characteristic radial distance between islands. Obviously, this model has its significance only in the proximity to the region of the rational surfaces discussed.

In the dimensionless variables, we have found that the dynamics again depend (besides the initial conditions) on the parameter $R = Am_p k_r^2 U_0 (Zq_p B_0^2)^{-1}$. For the numerical simulation, we chose $a = 2\,\mathrm{m}$, $R_0 = 3\,\mathrm{m}$, and we assumed that the system of islands will be localized close to $r = 1\,\mathrm{m}$, with $\lambda_r = 0.1\,\mathrm{m}$ and the poloidal wave number $m = 6$.

Figure 3 presents the system of separatrices, figures 4a ($R = 1$), 4b ($R = 5$) present the difference between the regular and chaotic behaviour (straight lines connect initial and final position of particles). We follow 10^3 particles situated in the proximity of the radius $r = 1\,\mathrm{m}$, and having, for simplicity, zero initial velocity. Here, in Fig. 4b, particles radially and poloidally traverse neigbouring magnetic islands. For real parameters $U_0 = 1\,\mathrm{kV}$, $B_0 = 2\,\mathrm{T}$, $\lambda_r = 0.1\,\mathrm{m}$, and for fully ionized carbon, the corresponding value is $R = 2.1 \times 10^{-2}$. For that value, our simple test-particle simulation offers no global diffusion, and from this point of view the proposed mechanism will not work. Nevertheless, a weak diffusion will always appear in the region close to the separatrices (as is confirmed by our PIC simulations). Moreover, ions with large Z_{eff} will transport a large amount of charge. This requires a detailed discussion by means of PIC simulations.

PIC simulation of the dynamics

In the present work we model impurity ion diffusion using the 2d3v PIC code XPDP2 from Berkeley with appropriate modifications [7].

Description of the model: In our 2D slab model the coordinates x and y correspond to the radial and poloidal directions, respectively. The magnetic field is directed along the z axis. In the poloidal direction the model is fully periodic. This means that all parameters are equal at the inner ($y = 0$) and outer ($y = L_y$) boundary. The particles crossing the inner or outer boundary are reinjected from the other boundary with the same velocity. In the radial direction, we have the following boundary conditions: at the right-hand (core plasma) side boundary ($x = L_x$), the are reflected; particles crossing the left-hand (wall) side boundary ($x = 0$) are removed. At $x = 0$ we inject Maxwell distributed particles assuming that outside the simulation region the density is n_0 and the temperature is T. In the slab, we apply the spatially periodical potential $U = U_0\,[\cos(kx + ky) + \cos(kx - ky)]$, decaying exponentially to zero in the radial direction at both boundaries. Diffusion of impurity ions can cause charge separation and a self-consistent electric field will emerge.

Plasma parameters: Impurity C^{1+}, density $n_i = 10^{16}\,\mathrm{m}^{-3}$, $B = 1\,\mathrm{T}$, impurity temperature $kT = 10\,\mathrm{eV}$, spatial period of the additional potential $\lambda = 10^{-2}\,\mathrm{m}$, $R = 0.9$.

Simulation parameters: Size of the system $L_x = 12cm$, $L_y = 6cm$, number of grid cells $N_x = 512$, $N_y = 256$, number of superparticles $N = 2.2 \times 10^6$. The time span was $20\,\mu$s.

Results: The following effects were found: The PIC simulation confirms the diffusion detected in the test-particle simulation. For the given parameters, a change in impurity density is found in the simulation region discussed. Concomitantly with this change, generation of a radial electric field of the order of $1\,\mathrm{kV/m}$ was detected (Fig. 5), which causes poloidal plasma rotation with a velocity of the order of $10^3\,\mathrm{m/s}$ (Fig 6). The diffusion was detected even for lower values of R than in the test particle model (e.g., for $R \approx 0.1$). The quasiperiodical change in the system parameters inside the simulation region is caused by the fact that the impurity ion density is larger in the valley regions. This is consistent with results of the test-particle simulation.

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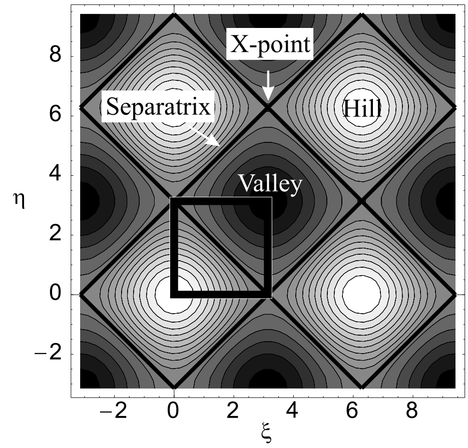

Fig.1. Equipotentials and separatrices.

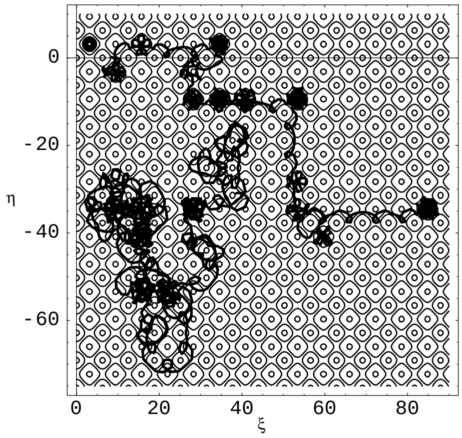

Fig.2. Examples of regular and chaotic motion.

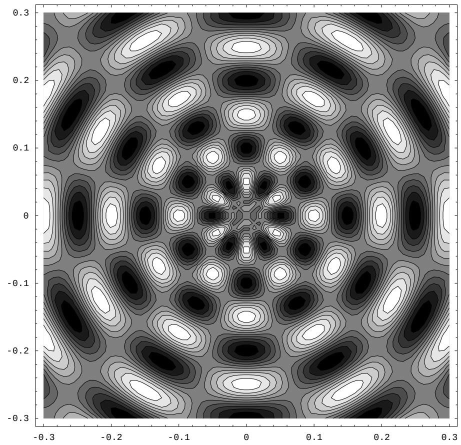

Fig.3. Equipotentials and separatrices in polar coordinates.

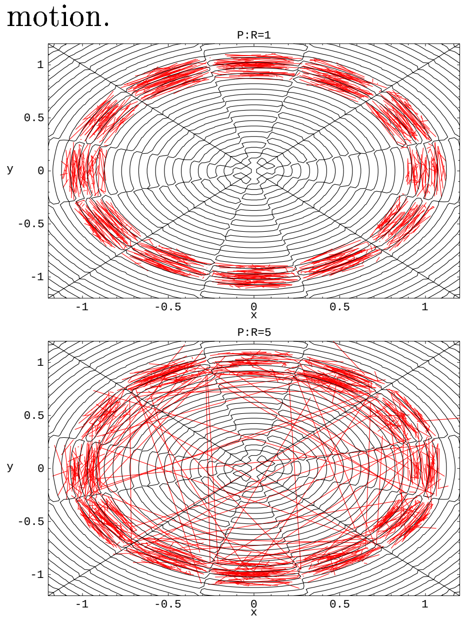

Fig.4. Regular and chaotic motion

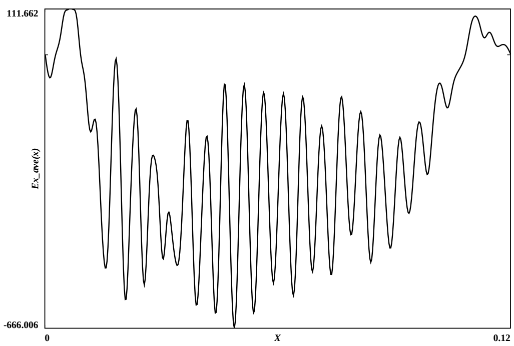

Fig.5. Radial electric field (V/m)

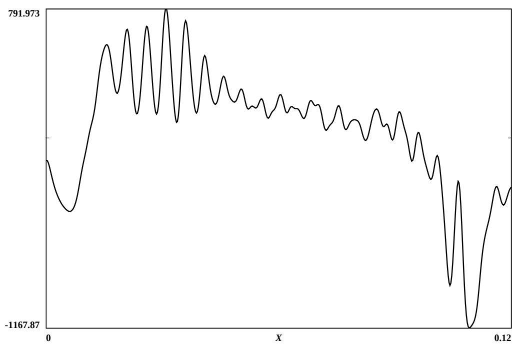

Fig.6. Poloidal velocity (m/s)